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**Parameter Optimization for  
Asynchronous Transfer  
Mode Leaky Bucket  
Policing Algorithm**

H. A. Abusalem  
C. J. Warner

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## INTRODUCTION AND BACKGROUND

Asynchronous Transfer Mode (ATM) promises bandwidth on demand with guaranteed quality of service (QoS) to transport and switch broadband integrated services in a statistical shared environment. QoS deals with specified cell-loss probability or cell-delay variation. It is an extremely difficult task to manage different traffic requirements on the same link. For example, a voice application sends data at low rates and is sensitive to transport delay and jitter while a text editing application is typically bursty, requires large buffers, and is not sensitive to transport delay variations. To ensure QoS, some traffic control mechanism is required, and in ATM this process is called Connection Admission Control (CAC). The ATM standards are still evolving and new control mechanisms are being considered for use in ATM. The current ATM policing and admission control standard is based on a continuous fluid flow model.

Our contention is that a discrete time analysis is more appropriate for packet networks and yields itself more readily to real-time dynamic applications. This work assumes an ATM environment where a negotiated contract is required to use the network. During connection setup, the user requests permission to enter the network and specifies his traffic class, which could be facsimile, file retrieval, or real-time video. The manager studies the user's traffic needs and checks if his bandwidth and buffering resources can accommodate an additional user without impacting the existing users. The manager grants permission with QoS guarantees. In return, the user is expected to honor his contract and not exceed his maximum peak rate and burst duration allowances. The network provider wants to protect the network from abuse and penalize violating users. The leaky bucket enforces the contract.

The leaky bucket resembles a token-credit scheme where each data packet requires a token to enter the network (i.e., a ticket is required to board the train). Based on user requirements, the provider sets the pace at which the tokens are generated and reserves a fixed-size token bucket. During slow and silent periods, tokens can accumulate in the bucket and any tokens that exceed the threshold, will be lost. This scheme slows the data and limits the maximum burst size that can enter the network. If a user violates and sends more packets and bursts more often, the packets have to wait in a data buffer until tokens become available and, eventually, will be lost if the data buffer becomes full.

ATM and fast links, in general, are very sensitive to policing reaction time. A misbehaving user can congest the whole network if the policing control is not fast enough. The leaky bucket is totally characterized by its depth (i.e., size) or threshold ( $B$ ) and token generation period ( $N$ ) that controls the leak rate ( $R$ ). We need to find the smallest  $R$  and  $B$  that satisfy certain QoS requirements. The advantage for controlling these parameters is that a small  $R$  exhibits better bandwidth efficiency and a small  $B$  is characterized by lower probability of congestion and reduced delay.

We will show that those are conflicting objectives. Many researchers are working on the area, but a clear grasp of how to make the proper tradeoff between  $R$  and  $B$  is not fully understood. Different traffic types require different guarantees. We analyzed the discrete leaky bucket system using the ON-OFF source model. We establish criteria to examine performance measures for different kinds of traffic in terms of cell-loss probability and queuing delays. We will also examine the scenario that, subject to a given limited bandwidth, the provider can allow two users with different QoS to share the bandwidth by determining the required dimensions for bandwidth and buffer sizes. We propose an optimization technique that will yield the optimum dimensions. We demonstrate these techniques with examples and illustrations. By maximizing network efficiency, the user will pay less and the

network will be able to service more users. The ATM services cost will greatly impact the success of ATM relative to competing gigabit technologies.

This report addresses two issues. The first is a complete analysis of the leaky bucket and development of performance measure criteria used in numerous supporting examples. The second focus is an optimization technique able to maximize the connection efficiency while satisfying a requested QoS by selecting suitable bandwidth and proper data buffer size.

## TRAFFIC MODEL

The traffic model is an ON-OFF bursty model widely used in the literature. The source transmits at its peak rate, defining a burst period extending over several active slots, and is off during the silent period. The source is characterized by the following parameters:

- $P_k$  = peak bit rate,
- $\rho$  = mean bit rate or source loading,
- $\beta$  = burstiness of a source is defined as peak over mean,
- $M$  = frame of consecutive ATM slots,
- $b$  = burst period.

The number of cells in each burst is geometrically distributed with mean burst length,  $b$ , and the OFF period is also geometrically distributed, and a frame consists of  $M$  consecutive ATM slots. Then this source at the start of each frame can be modeled as a two-state Markov chain with source transition probability matrix given by  $P_s$ :

$$P_s = \begin{bmatrix} P_{off} & 1 - P_{off} \\ 1 - P_{on} & P_{on} \end{bmatrix}$$

$$b = \frac{1}{1 - P_{on}} \quad \text{and} \quad \rho = P_k \cdot \frac{1 - P_{off}}{2 - P_{on} - P_{off}}.$$

The ON and OFF probabilities are expressed in terms of  $\rho$ ,  $b$ , and  $P_k$  as

$$P_{on} = 1 - \frac{1}{b}$$

$$\text{and } P_{off} = 1 - \frac{\rho}{P_k - \rho} \frac{1}{b}.$$

The source model is a discrete-time Markov Chain, called a phase process. It characterizes the state of the arrival process. The distribution of the number of packets that arrive during a slot is dependent on the phase of the arrival process and the distribution of the number of arrivals per slot given that phase.

## LEAKY BUCKET MODEL FORMULATION

We consider a leaky bucket system that consists of a single input line with a data buffer (D), a token bucket of maximum depth (B) and a single output line as shown in figure 1.

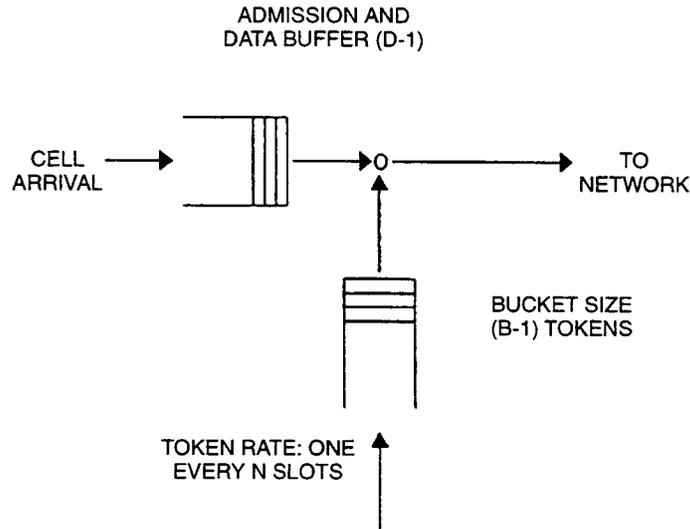


Figure 1. System model.

We model the leaky bucket after the continuous fluid flow model approach found in Onvural (1995). The source cell will be presented to the network only if a token is available in the bucket. For each token used, the bucket depth is decremented by one. If the token bucket is empty, the cell waits in the data buffer until a token becomes available. A token is generated at a deterministic rate every  $N$  ATM slots. The ATM slot is defined as the ratio between a cell size in bits (53 8-bit bytes) and the connection peak rate; for example, the slot length is  $3.7 \mu\text{sec}$  to transit a 424-bit cell by a 150-Mbits/sec connection.

We want to establish some criteria to study the leaky bucket performance. Our objective is to analyze the system at steady state or equilibrium. Typically, the QoS parameters are measured by the cell-loss probability, cell buffer waiting delay, and token-loss probability.

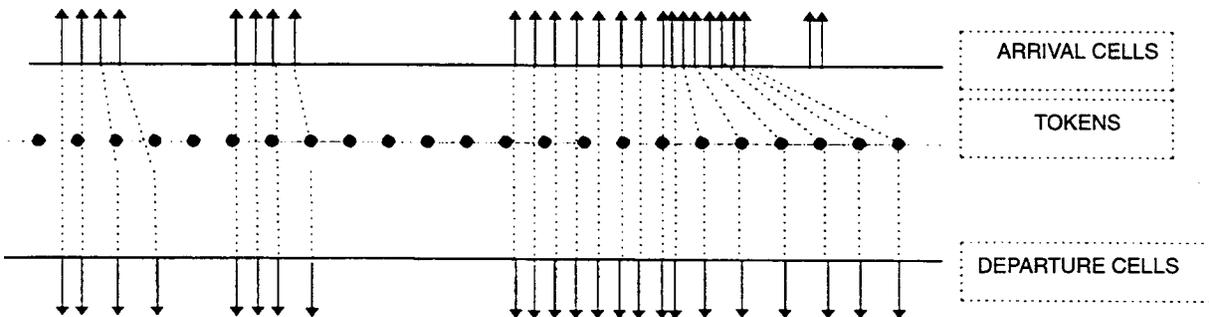


Figure 2. Time line for bursty traffic arriving and departing cells along with token presentation times.

For this section, we define the following variables:

- D is the data buffer for incoming cells.
- B is the token bucket for generated tokens.
- $P_{loss}^{cell}$  is the cell-loss probability where the cell is lost if the data buffer,  $D$ , is full.

$P_{loss}^{token}$	is the token-loss probability where the token is lost if the token bucket, $B$ , is full.
$I_E$	is an indicator function that takes a value of 1 for an event, $E$ , in a certain range of interest, and 0 otherwise.
$R$	is the output leak rate and the inverse interdeparture time for cells.
$\bar{b}_n$	is the number of tokens waiting in token pool at slot $n$ , $0 \leq \bar{b}_n \leq B$ .
$Burst_{max}$	is the maximum burst size that can enter the network.
$\bar{d}_n$	is the number of cells in the data buffer at slot $n$ , $0 \leq \bar{d}_n \leq D$ .
$\bar{q}_n$	is a discrete parameter denoting the difference between the number of cells in the data buffer and the number of tokens in the token buffer.
$\bar{a}_{ij}$	is a random variable phase arrival dependent Markov chain process where $i$ is the occupancy level and $j$ is the phase.
$\phi$	is the phase of the arrival.
$N$	is the token generation rate.
$p_s = \{p_{ij}\}$	is the source probability transition matrix and is the set of probabilities that the arrival process moves from phase $i$ to phase $j$ during a single slot, given no arrivals in that slot.
$P = \otimes_r P_S^R$	is the probability transition matrix that is the Kronecker product of each source probability transition matrix if there is more than one source.
$\pi_{i\phi}$	is the stationary (or equilibrium) distribution vector of $P$ , which signifies there are $i$ cells in the queue and the arrival process is in phase, $\phi$ , upon departure of the cell.
$Q$	is the continuous rate matrix.
$e$	is a column vector whose elements are equal to unity.
$I$	is an identity matrix whose diagonal elements are equal to unity.
$T_n$	is the periodic token generation rate between $0 \leq T_n \leq N - 1$ .

The leaky bucket system is a quadruple parameter system,  $(\bar{d}_n, \bar{b}_n, T_n, \phi)$ , and is totally described by its depth,  $B$ , and its token generation rate,  $N$ , which controls the leak rate,  $R$ . The system model is described by the following evolution equations:

$$\begin{aligned}
T_{n+1} &= (T_n + 1) \bmod (N), \\
\bar{d}_{n+1} &= \min \left( \max \left( (\bar{d}_n - 1 + I_{x_n=0}), 0 \right) + \bar{a}_{n+1}, D - 1 \right), \\
\bar{b}_{n+1} &= \min \left( \max \left( (\bar{b}_n - 1 + I_{Q_n=0}), 0 \right) + \bar{a}_{n+1}, B - 1 \right).
\end{aligned}$$

Onvural (1995) showed the system can be reduced to a triple system,  $(\bar{q}_n, T_n, \phi)$ , where  $\bar{q}_n$  is a new single-state variable,  $0 \leq \bar{q}_n \leq D + B$ , by mapping

$$\bar{q}_n = B - \bar{b}_n + \bar{d}_n.$$

With this mapping,

$$\begin{aligned}
\bar{q}_n &= 0 \text{ when } \bar{d}_n = 0 \& B = \bar{b}_n. \text{ And } \bar{q}_n = D + B \text{ when } \bar{b}_n = 0 \& D = \bar{d}_n, \\
\bar{q}_n &= \bar{d}_n - \bar{b}_n + (D + B - 2), \\
\bar{q}_{n+1} &= \max \left( (\bar{q}_n - 1 + I_{x_n=0}), 0 \right) + \bar{a}_{n+1}.
\end{aligned}$$

In steady state,  $\bar{q}_n = \bar{q}$  is not dependent on time.  $\pi = \{\pi_{01}, \pi_{02}, \pi_{12}, \dots, \pi_{(D+B)1}, \pi_{(D+B)2}\}$  is the steady-state, buffer-length distribution vector, and  $\pi_l = \{\pi_{l1}, \pi_{l2}\}$  is  $[1 \times k]$  elements of  $\pi$ , the probability that  $l$  cells are in the buffer at various arrival states and that the cell departs in phase  $\phi$ . The system is said to be at level  $l$  when the buffer occupancy at a departure is equal to  $l$ .

Observe, once  $\pi$  is known, the cell-loss probability, the leak rate, and expected delay can be found.

The token-loss probability is

$$P_{loss}^{token} = \pi_{01} + \pi_{02}.$$

The cell-loss probability is

$$P_{loss}^{cell} = 1 - \frac{1 - P_{loss}^{token}}{\rho \cdot N},$$

where  $\rho \cdot N$  is the token generation rate, the arrival source loading times  $N$  the token generation interval.

The leak rate is  $R = \rho(1 - P_{loss}^{cell})$ .

The mean number of slots between cell interdeparture time is

$$(R)^{-1} = (\rho(1 - P_{loss}^{cell}))^{-1}.$$

The number of cells waiting in the data buffer is

$$L = \sum_{i=B+1}^{D+B} (i - B)(P_{i1} + P_{i2}).$$

How do we solve for  $\pi$ ?

The steady-state (or equilibrium) solution of the analog homogeneous linear system,

$$\begin{aligned} \pi Q &= 0 \\ \text{and } \pi \cdot e &= 1, \end{aligned}$$

is presented by Onvural (1995). The steady-state discrete homogeneous solution satisfies the linear equations,

$$\begin{aligned} \pi &= \pi P \\ \text{and } \pi \cdot e &= 1. \end{aligned}$$

There are several techniques to solve the steady-state buffer distributions. The trivial solution is to use  $\pi(n) = \pi(n+1)P$  recursively until the Euclidian distance between any two vectors,  $\pi(n)$ ,  $\pi(n+1)$ , reach a specified minimum. The other solution is based on the special form of transition matrix,  $P$ , detailed in Neut (1981). We will show two techniques, one is based on matrix geometry solutions, and the other, on using spectral decomposition. In some instances, one technique is easier

than the other, or yields to a closed-form solution. For both techniques, we need the probability generating functions. Kleinrock (1974) has a lengthy development linking all those techniques.

In both techniques, we need the discrete Z transform or the probability generating function. Recall that the generating function is a mapping from a random variable to a power series equivalent to the Z transform.

### Solution Via Generating Function Probabilities Approach

The probability generating function is defined for a non-negative integer random variable  $\bar{a}$  by:

$$F_{\bar{a}}(z) = \sum_{i=0}^{\infty} z^i P\{\bar{a} = i\} \triangleq e[z^{\bar{a}}].$$

$F_{\bar{q}}(z) = \sum_{i=0}^{\infty} z^i \pi_i$  is the probability generating function for the steady-state  $\pi$  distribution.

$F_{\bar{a}}(z)$  is the diagonal matrix where the  $i$ th element is the probability generating function of the arrival process.

$F_{\bar{a}}^{(1)}(1) = \left. \frac{dF_{\bar{a}}(z)}{dz} \right|_{z=1}$  is the derivative with respect to  $z$  as  $z \rightarrow 1$ . It is also the first moment or mean.

$A(z) = P F_{\bar{a}}$  is defined as the arrival process.

Our objective is to develop a closed-form expression for the mean queue length,  $E[\bar{q}]$  for the slotted system having phase-dependent arrivals and unit service time.

$P_{ij} = P\{\bar{q}_{n+1} = j | \bar{q}_n = i\}$  is called the one-step transition probability. Upon conditioning on  $\bar{q}_n$ ,

$$\begin{aligned} P\{\bar{q}_{n+1} = j\} &= \sum_{i=0}^{\infty} P\{\bar{q}_{n+1} = j | \bar{q}_n = i\} P\{q_n = i\}, \\ P\{\bar{q}_{n+1} = j | \bar{q}_n = i\} &= P\{\bar{a}_n = j - \max((i-1), 0)\}, \\ \pi_j &= \lim_{n \rightarrow \infty} P\{\bar{q}_n = j\} \quad \pi = [\pi_0, \pi_1, \dots] \end{aligned}$$

We use several properties developed by Neut in 1981, (Theorem 1.2.1, page 10-11). The theorem says that if a Markov chain is positive recurrent, then we have the following properties:

$\pi_{i+1} = \pi_i K \geq 0$  and the eigen values lie inside the unit circle. Also, if the matrix

$$A(k) = \sum_{k=0}^{\infty} K^k A_k,$$

then  $\pi_0$  is left invariant eigen vector of  $A(K)$  normalized by

$$\pi_0(I - K)^{-1}e = 1.$$

$F_{\bar{q}}(1)$  is the vector at equilibrium solved from

$$F_{\bar{q}}(1) = F_{\bar{q}}(1)P = \frac{1}{N}\pi^T$$

$$\text{and } F_{\bar{q}}(1)_e = 1$$

The source does not transmit only in one state, state zero,

$$\pi_0 = \frac{1 - N\rho}{N} [1, 0, \dots, 0].$$

For stability conditions  $N\rho < 1$  because the average arrival per slot should be less than  $1/N$  the average number of tokens per slot.

Next, we derive equations (1) and (2) and (3) and equate the right-hand sides of equations (2) and (3) to find the  $E[\bar{q}]$ .

From standard  $Z$  transform techniques,

$$F_{\bar{q}}(z)[Iz - PF_{\bar{a}}] = \pi_0[z - 1]PF_{\bar{a}},$$

we differentiate both sides with respect to  $z$  to get equation (1):

$$F_{\bar{q}}(z)[I - PF_{\bar{a}}^{(1)}(z)] + F_{\bar{q}}^{(1)}(z)[Iz - PF_{\bar{a}}(z)] = \pi_0[z - 1]PF_{\bar{a}}^{(1)}(z) + \pi_0PF_{\bar{a}}(z). \quad (1)$$

Next, we take the limit as  $z \rightarrow 1$ , we get

$$F_{\bar{q}}^{(1)}(1)[I - P] = \pi_0P - F_{\bar{q}}(1)[I - PF_{\bar{a}}(1)].$$

Then we multiply both sides by  $e$  to find the marginal probability,  $\rho$ , that the system is not empty at the end of the slot,

$$\rho = F_{\bar{q}}(1)F_{\bar{a}}^{(1)}(1)e.$$

The next equation.....(2) uses that fact,

$$e F_{\bar{q}}(1)[I - P + e F_{\bar{q}}(1)] = e F_{\bar{q}}(1).$$

We use three operations to get the next equation (add to both of equations the term  $F_{\bar{q}}^{(1)}(1)e F_{\bar{q}}(1)$ , solve for  $F_{\bar{q}}^{(1)}(1)$ , and then post multiply by  $P F_{\bar{a}}(1)$ :

$$F_{\bar{q}}^{(1)}(1)P F_{\bar{a}}(1)e = F_{\bar{q}}^{(1)}(1)e\rho + \left\{ \pi_0P - F_{\bar{q}}(1)[1 - F_{\bar{a}}^{(1)}(1)] \right\} [1 - P + e F_{\bar{q}}(1)]^{-1}. \quad (2)$$

We get equation ...(3) by first differentiating equation ..(1) with respect to  $z$ , post multiplying by  $e$ , and taking the limit of both sides as  $z \rightarrow 1$ , and arrange

$$F_{\bar{q}}^{(1)}PF_{\bar{a}}^{(1)}(1)e = F_{\bar{q}}^{(1)}(1)e - \frac{1}{2}F_{\bar{q}}^{(1)}(1)F_{\bar{a}}^{(1)}(1)e - \pi_0PF_{\bar{a}}^{(1)}(1)e \quad (3)$$

to equate equations (2) and (3):

$$\begin{aligned} F_{\bar{q}}^{(1)}(1)e - \frac{1}{2}F_{\bar{q}}^{(1)}(1)F_{\bar{a}}^{(1)}(1)e - \pi_0PF_{\bar{a}}^{(1)}(1)e \\ = F_{\bar{q}}^{(1)}(1)e\rho + \left\{ \pi_0P - F_{\bar{q}}^{(1)}(1) \left[ 1 - F_{\bar{a}}^{(1)}(1) \right] \right\} \left[ 1 - P + eF_{\bar{q}}^{(1)}(1) \right]^{-1}. \end{aligned}$$

Finally, solve for  $F_{\bar{q}}^{(1)}(1)e$ :

$$\begin{aligned} E[\bar{q}] = F_{\bar{q}}^{(1)}(1)e = \frac{1}{1-\rho} \left\{ \frac{1}{2}F_{\bar{q}}^{(1)}(1)F_{\bar{a}}^{(2)}(1)e + \pi_0PF_{\bar{a}}^{(1)}(1)e \right. \\ \left. + \left( \pi_0P - F_{\bar{q}}^{(1)}(1) \left[ 1 - F_{\bar{a}}^{(1)}(1) \right] \right) \times \left[ 1 - P + eF_{\bar{q}}^{(1)}(1) \right]^{-1} PF_{\bar{a}}^{(1)}(1)e \right\}. \end{aligned}$$

### Approximate Solution Via Spectral Decomposition

From continuous flow theory referenced by Acampora (1994), the cell-loss probability is approximated by the following buffer overflow probability:

$$p(\text{No. of cells in buffer} > x) = ke^{z_0x},$$

where  $z_0$  is the largest negative eigenvalue of the steady-state transition probability matrix, and  $k$  is a constant independent of the buffer size,  $x$ . Because  $z_0$  dominates the behavior of the cell probability, the cell-loss probability is given by  $p_x(\text{cell loss}) = k_{z_{\max}^x}$ . The constant equals the source loading,  $k = \rho$  for the finite buffer case, and for our geometric ON-OFF source, the approximate solution involves solving for the roots of a quadratic equation. Solving for the largest eigen value inside the unit disk is harder than finding the smallest positive root outside the unit circle.

$\Delta(z) = |zI - A^N(z)|$  is the determinant value and  $\lambda_1$  &  $\lambda_2$  are the eigen values:

$$\Delta(z) = (z - \lambda_1^N(z))(z - \lambda_2^N(z)).$$

From standard matrices techniques:

$$\begin{aligned} |zI - A^N(z)| &= |zI - A(z)|, \\ A(z) &= \begin{bmatrix} P_{\text{off}} & 1 - P_{\text{off}} \\ z(-P_{\text{on}}) & z(P_{\text{on}}) \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} A^N(z) &= a_0(z)I + a_1(z)A(z), \\ a_0(z) &= \frac{\lambda_1(z)\lambda_2(z) \left[ \lambda_2^{N-1}(z) - \lambda_1^{N-1}(z) \right]}{\lambda_1(z) - \lambda_2(z)}, \\ a_1(z) &= \frac{\lambda_1^k(z) - \lambda_2^k(z)}{\lambda_1(z) - \lambda_2(z)}, \end{aligned}$$

$$\lambda_{1,2}(z) = \frac{[p_{off} - p_{on}]}{2} \pm \frac{\sqrt{(p_{off} + zp_{on})^2 - 4z(p_{off} + p_{on} - 1)}}{2},$$

$$z_{\max} = \lambda_1^N(z).$$

When we use laurent series expansions, we can get an approximate solution in terms of the burst period and the source loading,

$$z_{\max} = 1 + \frac{1}{b} \cdot \frac{N(1 - N\rho)}{(N - 1)(1 - \rho)} + \frac{1}{b^2} \frac{N(1 - N\rho)(-1 + 2N - N^2\rho)}{2(N - 1)^2(1 - \rho)^2} + O\left(\frac{1}{b^3}\right),$$

$$z_{\max}^{-1} = 1 - \frac{1}{b} \cdot \frac{N(1 - N\rho)}{(N - 1)(1 - \rho)} + \frac{1}{b^2} \frac{N(1 - N\rho)(1 - N^2\rho)}{2(N - 1)^2(1 - \rho)^2} + O\left(\frac{1}{b^3}\right),$$

We assume cells are lost if a frame,  $M$ , exceeds the buffer capacity,  $x$ , therefore, the first term in the expansion is equivalent to the cell-loss probability:

$$p_x(\text{cell loss}) = \rho \cdot z_{\max}^M \left[ 1 - \frac{1}{b} \cdot \frac{N(1 - N\rho)}{(N - M)(1 - M\rho)} \right]^{x-M}.$$

## Examples and Results

The bucket depth can have a significant effect on the packet waiting time. As the normalized token generation rate shown on the abscissa of figure 3 varies from 0 to 1, the actual token generation rate varies from the average cell rate to the maximum burst cell rate of the data source. Figure 3 shows that when the token generation rate is near the average cell rate, there is little variation of expected waiting time and independent of the token buffer size. At the other extreme, when the token generation rate approaches the peak burst rate, there is significant reduction in cell waiting time for the shorter token buffers. It is this reduction in expected waiting time we seek by using shorter token buffers. We demonstrate in figure 4 how the cell-loss probability is affected by the normalized source loading factor (ratio of mean to peak rate) for different token buffer depths. In this figure,  $D$ , the data buffer depth is zero. We note that as the loading factor approaches unity, loss probability approaches a constant 0.10, which matches the token delivery rate of 1 token per 10 ATM slots. By way of comparison, the cell-loss probability as a function of normalized source loading for different length data buffers is essentially the same curve set as in figure 3. We can thus conclude that the length of data buffer and the length of token buffer have identical effect on cell-loss probability, and that examining the effect of token buffer length adequately accounts for the effects of either buffer. Subsequent analysis and presentations will treat the token bucket as a data buffer of varied depth.

For the next set of examples, we examine the cell-loss ratio as a function of cell leak rate for different values of bucket depth. We assume a constant leak rate as the throughput of the system and ignore the delay to show the tradeoff between the bucket size and the leak rate. We find the cell probability function is coupled with both the leak rate and the bucket size.

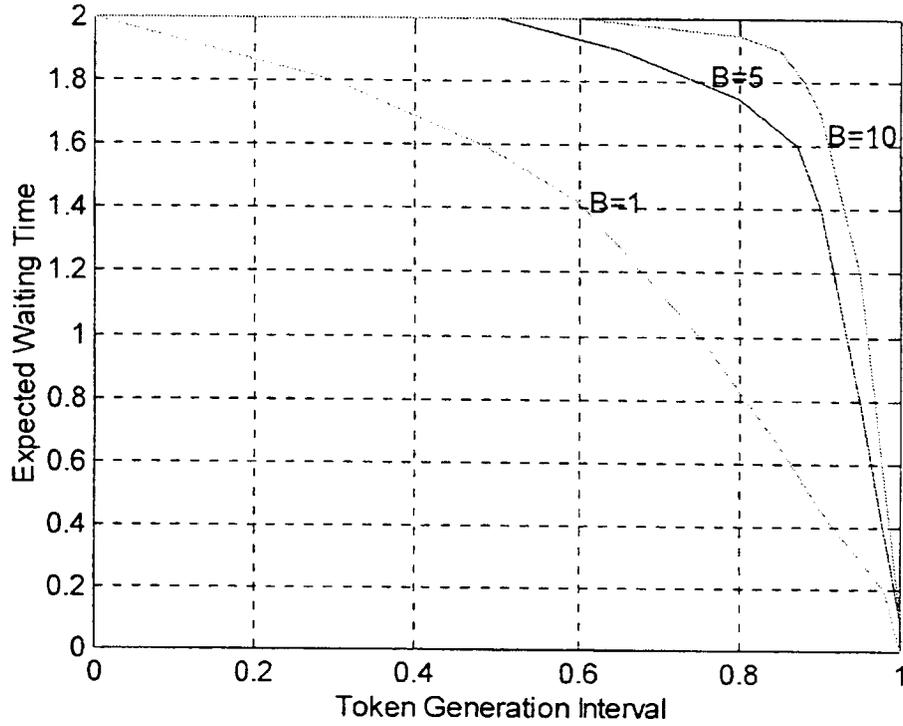


Figure 3. Expected cell waiting for different leaky bucket sizes.

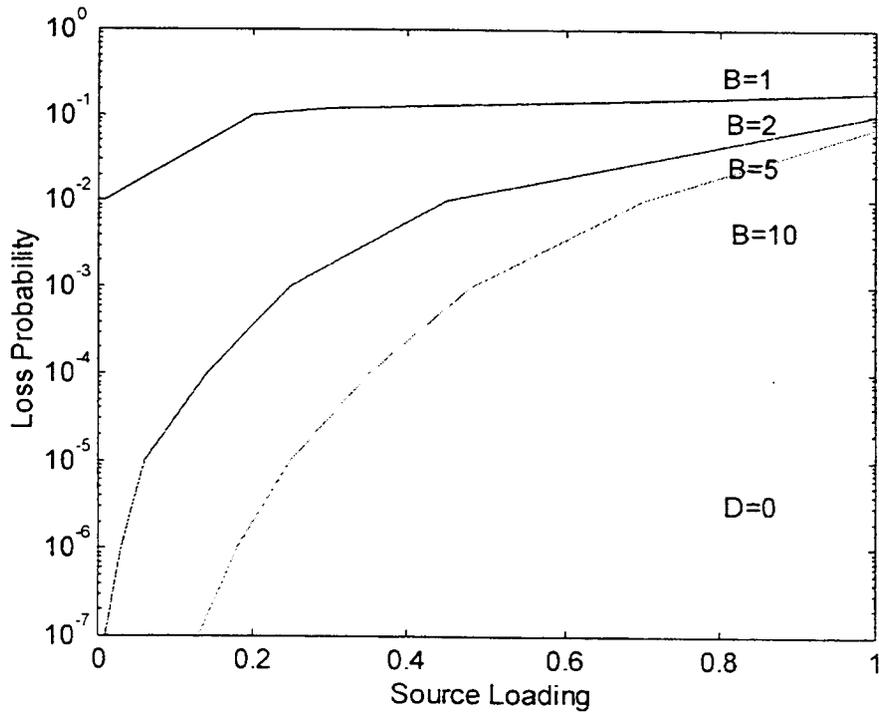


Figure 4. Source loading vs. CLP for zero data buffer.

Figure 5 shows for a bursty video source, increased buffering is required to obtain a lower cell-loss ratio. Note that the burstiness is defined as the ratio between the peak and mean input rates. For a cell-loss ratio of 0.1, the bucket size must be 200 to obtain the leak rate of 35. Note that the curves corresponding to  $b = 100$  and 200 have two converging components. The lower of these was obtained by simulation and the upper of these was obtained by a closed-form exact solution. A 10 percent cell-loss ratio is outrageously high. To realize any realistic cell-loss ratio, the system would require extremely large data buffers. We conclude, as have many others, that the leaky token bucket does not serve as a smoothing (or shaping) process, and at most, can only serve as a contract policing function. An additional undesired consequence of the large buffer requirement is greater cell delay. Figure 6 presents results similar to figure 5. Here the abscissa of the figure is the bucket size, and the parameter identifying the operating condition is the buffer leak rate. Rates close to 10 percent were selected since it is desirable to operate in the vicinity of the mean leakage rate. Figures 5 and 6 were generated using an optimum search technique that satisfies a certain cell-loss probability where the search region is bounded by the mean and peak rate. Note that for the buffer sizes 100 and 200 we find the simulation results are very close to the numerical analysis results.

We now address the interaction between leak rate, bucket size, and maximum burst rate. Figure 7 presents curves of the buffer size required to sustain maximum burst rates as a function of data leak rate. As figure 7 shows, the larger the buffer, the larger the maximum burst size allowed to enter the network. The CLP that is assumed in this section is  $10^{-9}$ . The abscissa is a normalized leak rate where zero corresponds to the leak rate equal to the sustained rate or average rate and one is equal to the peak rate.

Figure 8 shows more clearly the linear relationship between the burst size and the buffer size by parameterizing the curves on leakage rate and plotting maximum burst rate against bucket size. Again, as the leak rate/peak ratio approaches one, the network can accommodate greater burst size, but will exhibit no network bandwidth efficiency.

In the above scenarios, we examined the performance of leaky bucket in terms of cell-loss probability, delays, and token generation. In figure 9 we examine the sensitivity of the leaky bucket to ON-OFF arrival model, which is actually the ATM standard for the arrival process.

Figure 9 shows the dramatic effects of the ON-OFF period. Note that the four curves have the same mean or source loading, which is 0.5. The number of sources is 10 for the (8–32) and (200–800) curves, and 50 users for the peak for (40–60) and (1000–4000) curves, respectively. It is obvious that we need to exercise caution in picking the geometric ON-OFF source as an arrival model.

## **CONNECTION ADMISSION EXAMPLE**

Figure 10 shows that a better statistical multiplexing is obtained if the traffic sources are bursty. In this example, we varied the traffic burstiness for the same load and buffer parameters is the statistical gain and network efficiency.

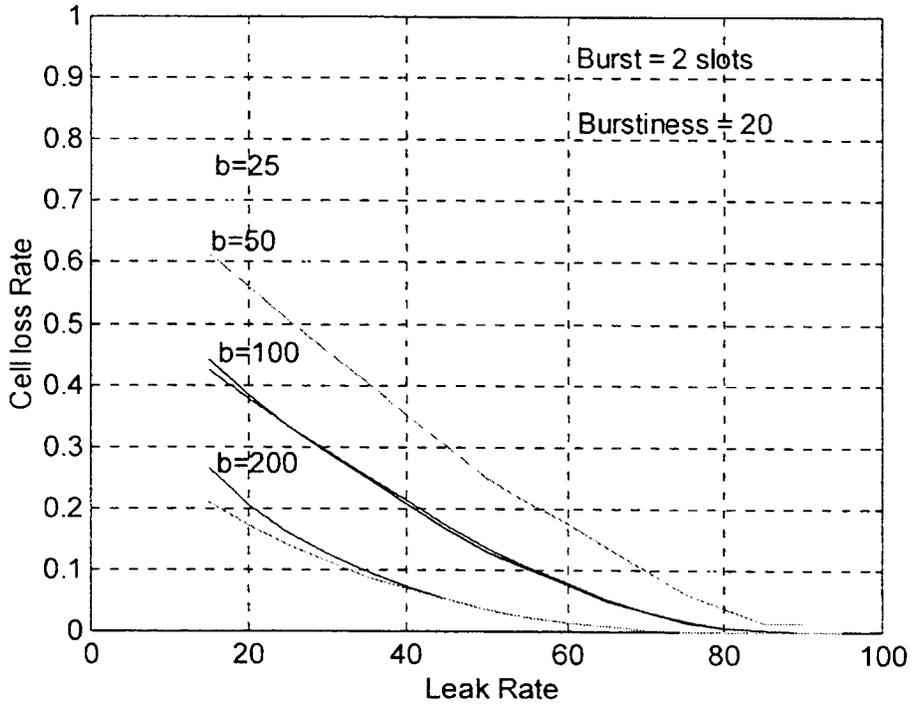


Figure 5. Analysis and simulation results.

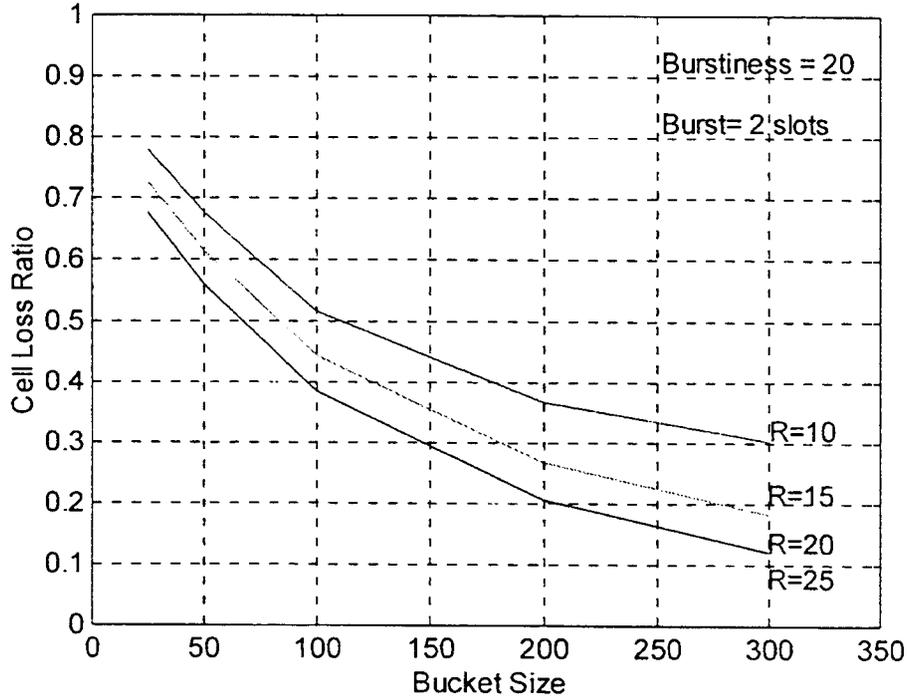


Figure 6. Trade-off between bucket size and leak rate.

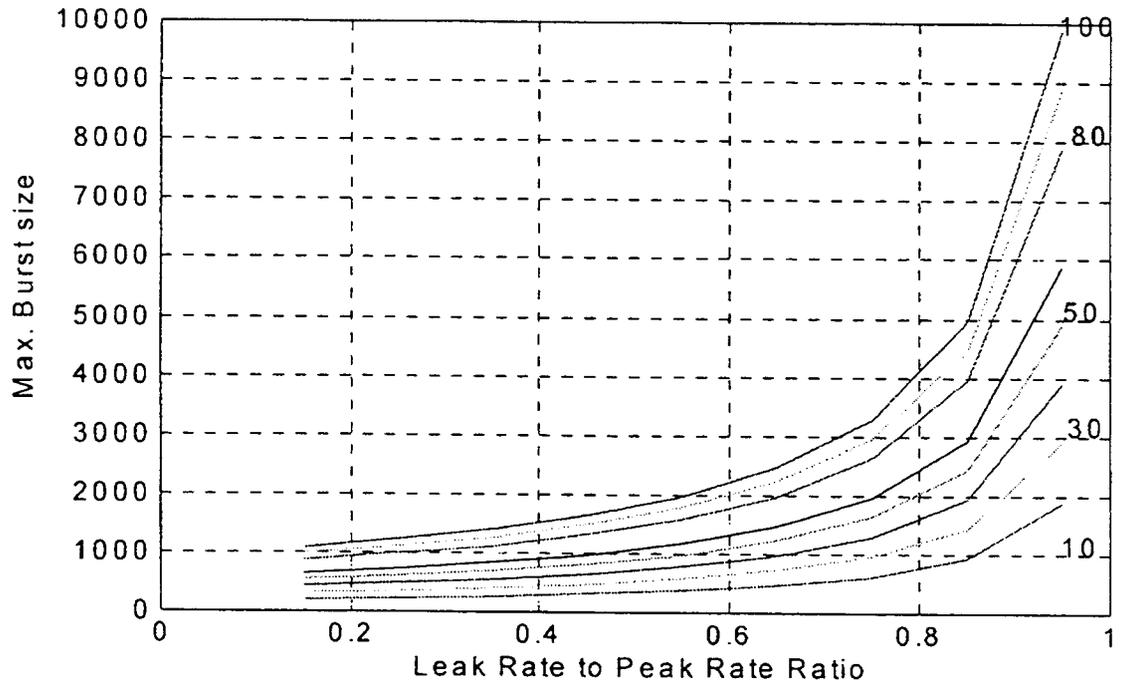


Figure 7. Maximum burst rate vs. leak rate.

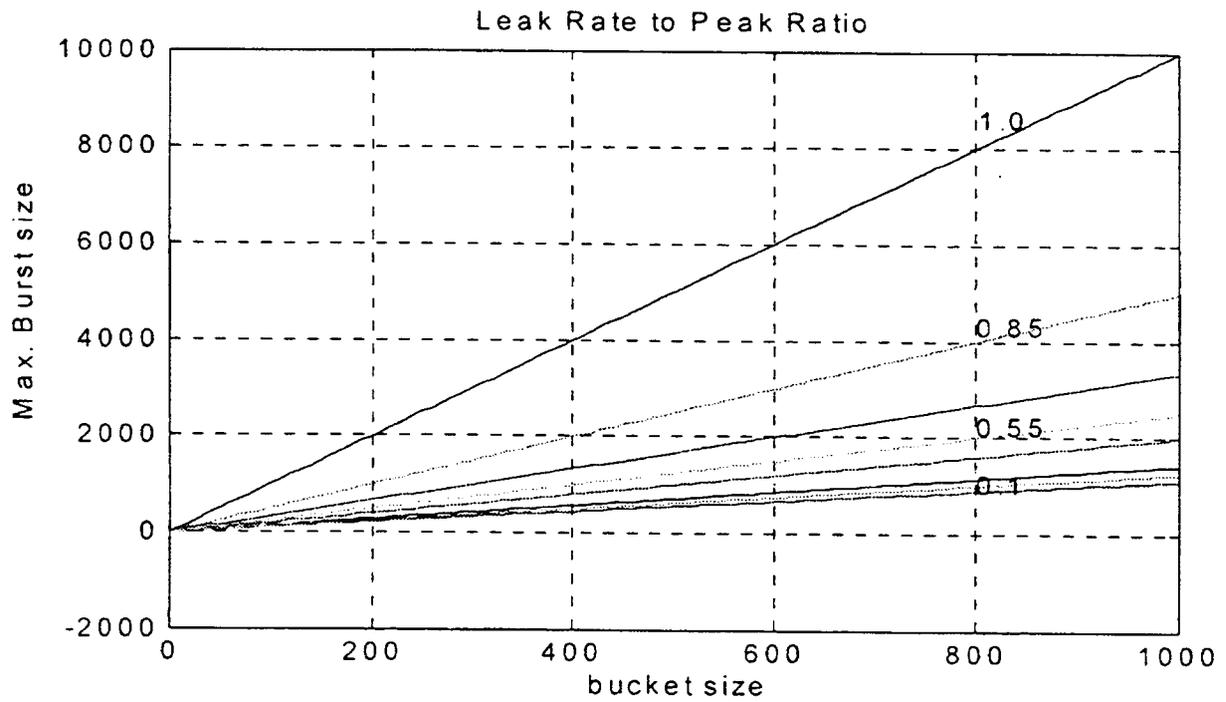


Figure 8. Effect of maximum burst size on leaky bucket parameters.

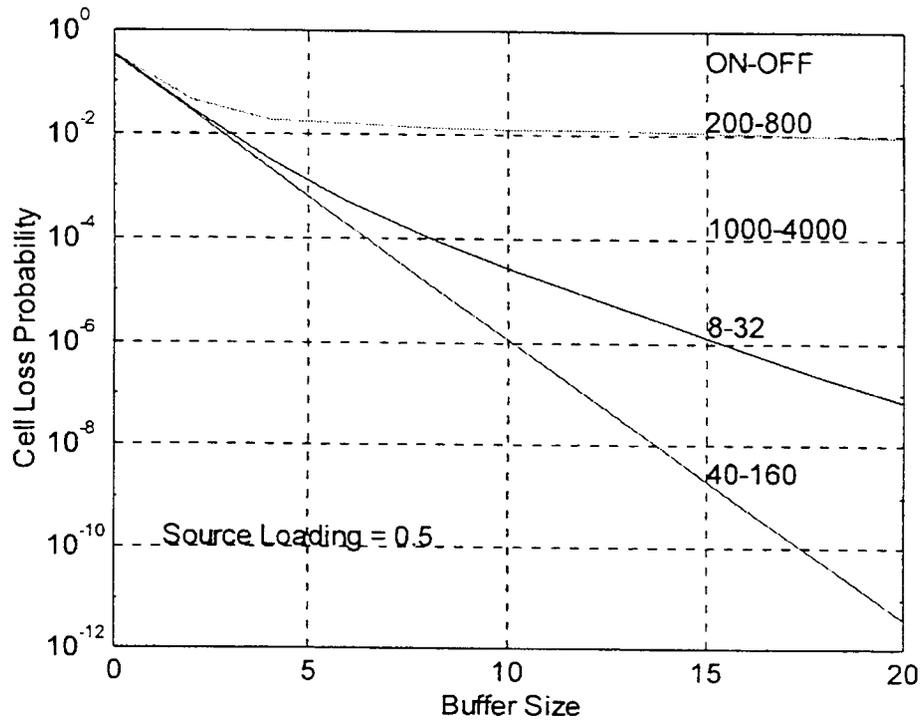


Figure 9. Effects of ON-OFF period.

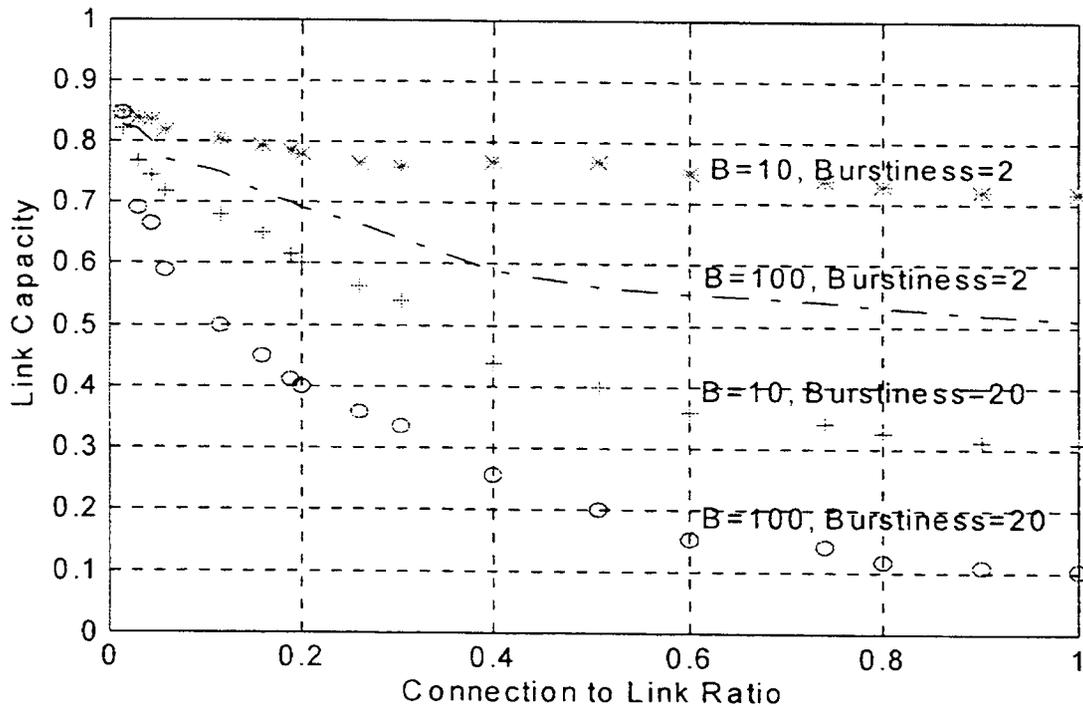


Figure 10. Admission control example.

**How Effective is the Dual Leaky Bucket ATM-Forum Admission Standard?** The ATM-forum UNI-3.0 standardized dual leaky buckets for policing control. The first leaky bucket had a unit depth and a leak rate that is the reciprocal of the peak rate to police constant bit rate traffic while the second leaky bucket has a depth that was defined by the maximum burst size, and the leak rate equals the reciprocal of the average rate, also called the sustained cell rate. The leaky bucket serves only as a tagging device for violating cells. In 1991, Butto and his colleagues studied the effectiveness of the leaky bucket policing mechanism by judging its performance according to cell-loss probability criteria as a function of arrival process parameters. They concluded that it is easy to control the peak rate, but to control the mean is a harder problem. It is a difficult task to control the mean bit rate because observing and controlling violating parameters needs to be averaged over a long period of time because in a fast network the reaction time depends heavily on the connection time of the source.

In this section, we study the feasibility to use dual leaky buckets. Table 1 shows a six-case study assuming a cell-loss probability of  $2 \times 10^{-6}$ . For the first two cases, the peak is 1/48 the link capacity. The token generation rate was set to 0.7 of the peak rate for the first bucket and 0.5 of the peak rate for the second bucket. It is clear that the average mean rate for these two cases is very close. Note the difference of the buffer requirements. The leaky bucket depth is mainly dependant on the silence period as the off period. In cases four and six, we chose peak rates of 1/20 and 1/10 and token generation rates of 0.67 and 0.77 of the peak rate, respectively. We find the bucket size requirements to be the same. From the above, we see the OFF duration controls the leaky bucket size. Examining the leaky bucket conditions of cases one and two, we again observe the trade-off between the token generation rate, ON-OFF sizes, and the required bucket size.

Table 1. Leaky bucket stage study results.

	Peak Rate	ON	OFF	1st LB R $10^{-3}$	1st LB B	1st LB R/P	2nd LB R $10^{-3}$	2nd LB Size B	2nd LB R/P
1	0.0208	22	40.624	13.9	118	0.667	7.6	2790	0.5
2	0.0208	110	203.12	13.9	589	0.667	7.6	13911	0.5
3	0.05	100	1000	33.3	420	0.667	4.7	25255	0.5
4	0.05	100	100	38.5	372	0.769	25.8	7380	0.625
5	0.01	100	1000	6667	420	0.667	0.9	25255	0.5
6	0.01	100	100	7692	372	0.769	5.163	7380	0.625

**How Can We Design An Efficient Network?** There is a current debate in the ATM community on how to maximize network efficiency. Do we optimize edge devices or add complexities to the switch? Our approach is to keep the switch simple and fast and provide the network manager with the necessary tools to achieve maximum utilization.

### Optimization of the Leaky Bucket Parameters

Network providers have the difficult task of guaranteeing a certain quality of service for their users. Consider the following call admission problem. Two users request service with normalized loads, QoS, requirements ( $R_1$  at  $QoS_1$  and  $R_2$  at  $QoS_2$ , respectively). The provider must allocate separate buffers and statistically multiplex the available bandwidth between the two users. What are the minimum network buffer,  $B_n$ , and bandwidth,  $C$ , resources that will meet the specified cell-loss probability and delay constraints for each service? We want to assign time slots to the two users. There

are three conditions for assigning a slot, when it is available, to the two users. For the first condition, neither user has a cell waiting in the input buffer and the time slot is left vacant because there is no traffic available to fill it. For the second condition, the input buffer contains a cell from only one user. When the time slot is made available, the cell from the single user is presented to that slot. For the third option, both users have cells in the input buffer. When a time slot is made available, the manager must select a cell from one of the users. We define  $P_r$  as the probability of assigning the time slot to user 1 and  $(1-P_r)$  as the probability of assigning the slot to user 2. We also define the steady-state probabilities that no cells are waiting in the buffers for sources 1 and 2 as  $P_{r1}(0)$  and  $P_{r2}(0)$ . Define  $P_{si}$  as the probability of assigning a time slot to the  $i$ -th user conditioned on a cell from the  $i$ -th user being available in the data buffer. Then we determine the effective conditional probability of assigning a time slot to user 1 and 2 approximately as

$$\begin{aligned} p_{s1} &= p_{r2}(0) + (1-p_{r2}(0))p_r, \\ p_{s2} &= p_{r1}(0) + (1-p_{r1}(0))(1-p_r). \end{aligned}$$

We observe that these conditional probabilities split the system into two loosely coupled queues with geometric arrivals and geometric service time, the Geo-Geo-1 queuing model. The stationary state probabilities are given by:

$$p_{ri}(j) = \left\{ \begin{array}{l} 1 - \frac{R_i}{P_{si}} \quad j = 0 \\ \frac{p_r(0)}{1-p_{si}} \left( \frac{R_i(1-p_{si})}{(1-R_i)p_{si}} \right)^j \& \frac{(1-p_r(0))}{1-p_{si}} \left( \frac{R_i(1-p_{si})}{(1-R_i)p_{si}} \right)^j \quad j > 0, i = 1, 2 \end{array} \right\},$$

$$p_{r1}(0) = 1 - \frac{R_1}{P_{s1}} \text{ and } P_{r2}(0) = 1 - \frac{R_2}{P_{s2}}.$$

Let  $B_n$  be the total network buffer size that is equal to the sum of  $B_1$  and  $B_2$ . Let  $q_i$  be the number of cells from user  $i$  offered to the buffer. The probability of cell loss for each user is expressed approximately as

$$p_{r_i}^{loss} = p_r\{q_1 > B_i\}P_r\{q_1 + q_2 > B_n\}R_1R_2 \quad i = 1, 2.$$

Recall that losses occur as the complementary queue function distribution,  $G_i(1$ -steady state function distribution),

$$\log p_{r_i}^{loss} = \log G_i(B_i) + \log G(B_1 + B_2) + \log (R_1R_2) \quad i = 1, 2.$$

Recall that in the steady state, the stationary probabilities satisfy  $\pi = \pi P$ , for which we had developed a closed-form solution. Losses occur when two services arrive and the system is full:

$$\begin{aligned} p_{r_1}^{loss} &= \left[ \pi(B_1 + B_2)(1-p_r) + \sum_{n=B_1+1}^{B_n} \pi(n, B_n - n) \right] R_1R_2, \\ p_{r_2}^{loss} &= \left[ \pi(B_1 + B_2)p_r + \sum_{n=B_2+1}^{B_n} \pi(B_n - n, n) \right] R_1R_2. \end{aligned}$$

Then, the approximate network cell-loss probability is

$$p_{r_n}^{loss} = p_{r_1}^{loss} + p_{r_2}^{loss} = P_r\{q_1 + q_2 > B_n\}R_1R_2.$$

The desired optimal network buffer size, i.e. space resource, is found as

$$B_n = \frac{\log(p_{r_1}^{loss} + p_{r_2}^{loss}) - \log R_1R_2}{\log(R_1R_2) - \log((1 - R_1)(1 - R_2))}.$$

Define  $D_i$  as the complementary probability delay distribution (the probability that the delay percentile,  $d_i$ , is less than  $k$  slots):

$$D_i(k) = 1 - p_r\{d_i < k\} = 1 - \sum_{n=0}^{B_n} \pi_k(0, n) \quad \text{and } \pi_k = \pi,$$

$$\log D_i(d_i) = d_i \log \frac{1 - P_{si}}{1 - R_i},$$

then its the slope,  $l_i, l_i = \frac{\log D_i(d_i)}{d_i}$ .

Define  $L_i = e^{l_i}$ , and multiply by the time resource,  $C$ , we get

$$CP_{si} = C(1 - L_i) + L_iR_i \quad i = 1, 2.$$

Substitute for  $p_r$  and set to zero,

$$p_r = \frac{L_1R_1 - L_2R_2 + R_1}{R_1 + R_2} + \frac{L_2 - L_1}{R_1 + R_2}C.$$

Then we find  $C$ , which is the positive root of this quadratic equation:

$$\left(L_2(1 - L_1)(1 - L_1) - R_1 \frac{L_2 - L_1}{R_1 + R_2}\right)C^2 + \left(L_1L_2(R_1 + R_2) - L_2R_2 - R_1 \frac{L_1R_1 - L_2R_2 + R_2}{R_1 + R_2}\right)C - L_1L_2R_1R_2 = 0.$$

Recall that the log of the complementary steady-state probability queue distribution function is given by  $\log(G_i(B_i)) = m_i + B_i k_i$ ,

where

$$k_i = \log\left(\frac{R_i}{P_{si}}\right) \& m_i = \log\left(\frac{R_i(1 - P_{s1})}{P_{si}(1 - R_i)}\right) be$$

Now, we are ready to find an optimal bucket or buffer size for service  $i$ ; we use the ratio,

$$\frac{p_{r_1}^{loss}}{p_{r_2}^{loss}} = \frac{\log(G_1(B_1))}{\log(G_2(B_2))} \quad \text{and } B_2 = B_n - B_1.$$

We now determine the desired optimum buffer size,  $B_1$ , as

$$B_1 = \frac{1}{m_1 m_2} + m_2 B_n + \log \left( \frac{p_{r1}^{loss}}{p_{r2}^{loss}} \right) + k_2 - k_1.$$

We now address the optimality criteria. Also, we study the interaction between delay specification, network load, and the priority or probability assignments:

$$\text{if we find a normalized load, } \gamma = \frac{R_1 + R_2}{C}, \text{ where } 0 \leq \gamma \leq 1.$$

It is obvious from the curves, that for a constant load, both delay-1 and delay-2 curves are a decreasing function. For a constant priority, both delays are an increasing function. Therefore, there is an optimum point for (load, priority) where the curves meet in figure 11. There are numerous optimizing techniques in literature to find the point of intersection.

The following numerical example will clarify the optimization scheme:

$$\begin{aligned} \text{user}_1 : D_1(9\mu s) &= 10^{-3} \text{ and } p_1^{loss} = 10^{-6} \\ \text{user}_2 : D_2(4\mu s) &= 10^{-3} \text{ and } p_2^{loss} = 10^{-9} \end{aligned}$$

For a given 1 Mbit/sec link capacity, the time slot is  $1 \mu/\text{sec}$ . The delay requirements for users 1 and 2 will be 9 and 4 slots, respectively. Observe that the area under the solid trapezium satisfies the users' cell-loss probability requirements. To optimize performance, the solid line should be moved to the dotted line to close the gap between that trapezium and the delay curve for a 1-Mbits line, as in figure 12.

For the users normalized loads of 0.2 and 0.4, the new delay slots that meet the specified QoS are 6.2 and 6.8, and the minimum bandwidth is 0.93 Mbits/sec. This frees 70 Kbits of bandwidth, which can easily support a low data rate application. We can calculate the buffer dimensions for the two users. Figure 13 shows that the new required buffers for users 1 and 2 are 6.1 and 6.9, and the total network buffers is 13 cells.

Figure 14 shows the new reduced delays that will satisfy the users cell-loss probability requirements. For user 1 with 0.25-priority, the new delay is 8.35 instead of 9 slots, and 3.72 instead of 4 slots for the 0.75 priority user.

We demonstrated a new technique to obtain optimum statistical multiplexed bandwidth usage and buffer allocation for two users with different cell-loss probability, delay, and time priority probability requirements. The technique could be easily extended to multiple users with significant degree of complexity.

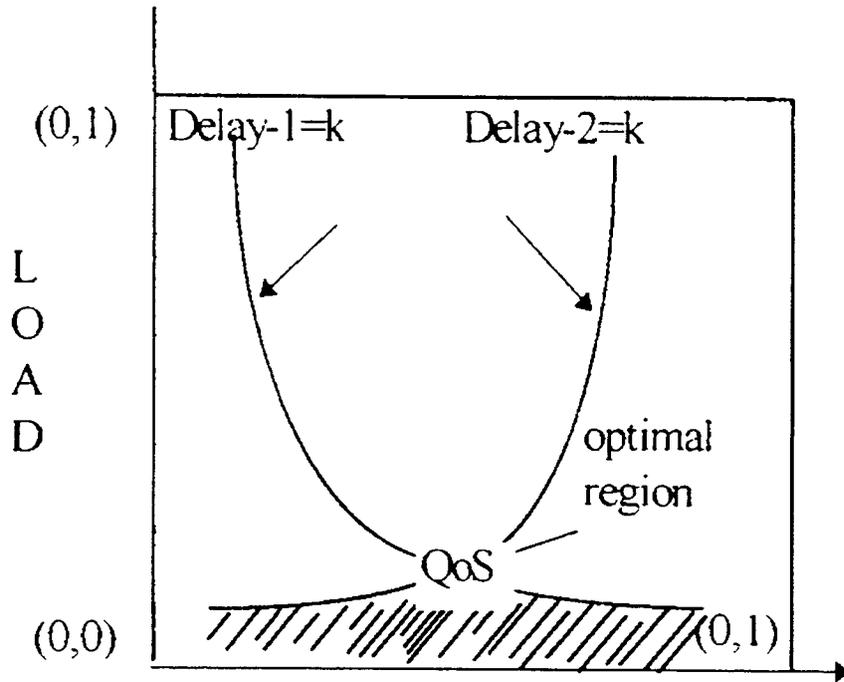


Figure 11. Load delay plane.

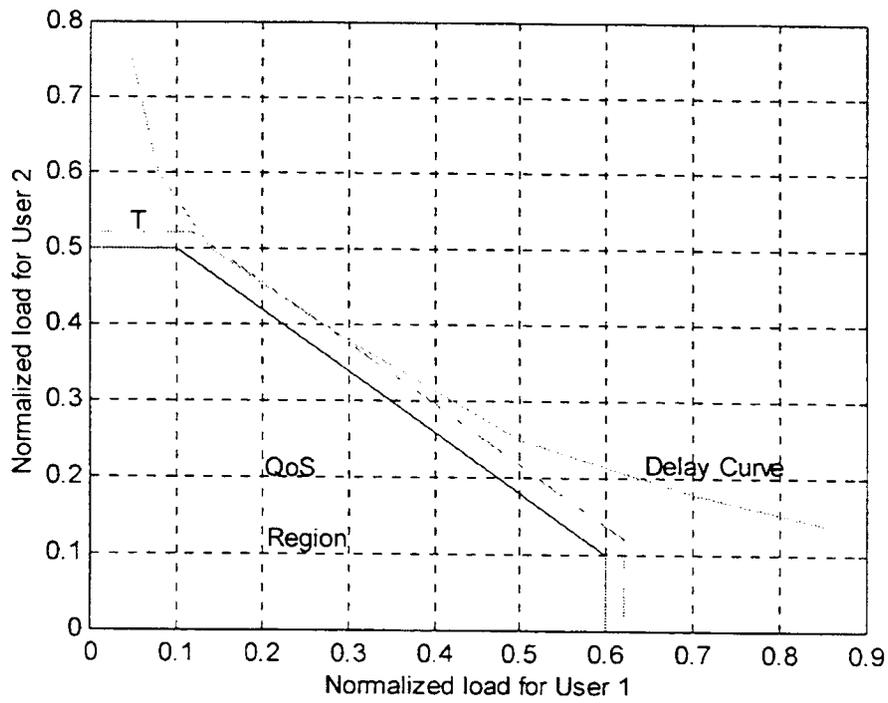


Figure 12. Disjoint region between capacity and QoS.

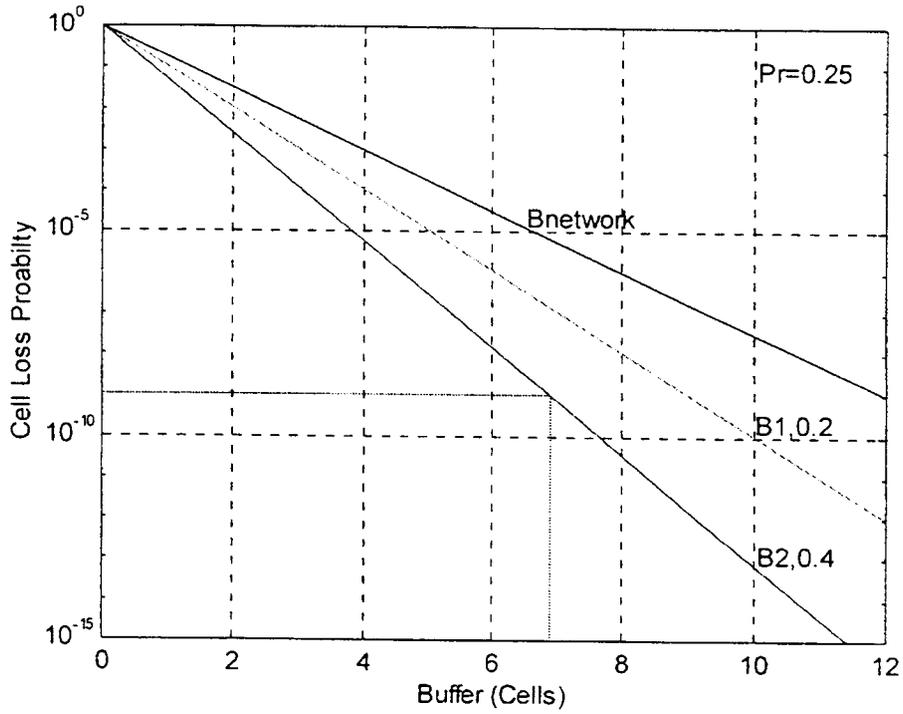


Figure 13. Buffer allocations for normalized loads 0.2 and 0.4.

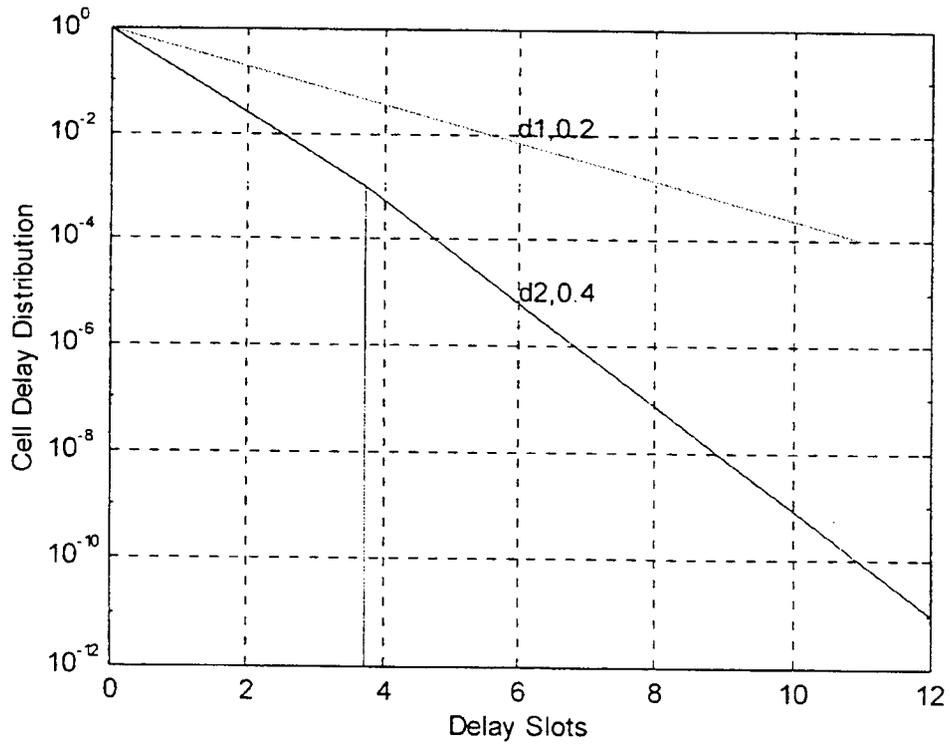


Figure 14. Delay requirements for normalized loads 0.2 and 0.4.

## CONCLUSIONS

We have developed two solutions for discrete leaky bucket policing algorithm to find cell loss probability and steady state buffer distribution. We applied the closed form solution to the task of determining the optimum length data buffer required to service two users. We followed an approach similar to that used in the analog model. We determined the effects of token generation, source loading on the cell loss probability. We showed the trade off between the different parameters of the leaky bucket and also the sensitivity of the leaky bucket to the parameters of the ON-OFF source model. We examined the performance of dual leaky buckets for different traffic classes. Finally, we devised an approach to optimize the required buffer size and buffer partition to service two users. This crucial information is required by the network manager. The approach taken was to optimize the total network resources in terms of bandwidth and buffer requirements and then allocate portions of the specified resources to each user.

A possible research topic, allied with this work, is the caliber and access goodness of fit of traffic models used to represent real traffic. This should include development of simple and accurate techniques to determine the parameters for assumed traffic models obtained by observing and processing real traffic offered to the network. The emphasis here is simple: As Gallager noted recently at a wireless conference, a simple model which is 90 percent accurate is better than a complicated model that is 91 percent accurate or worse bears little resemblance to the real world.

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