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SHF SATCOM Terminal Ship-Motion Study

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EXECUTIVE SUMMARY

OBJECTIVE

This study was initiated to support development of the Phase-III Super-High Frequency (SHF) SATCOM Terminal Specification by ensuring that it incorporates and specifies all relevant characteristics related to ship motion. The purpose of the specification is to define the realistic requirements that ship motion imposes on the SHF SATCOM Terminal under operational conditions.

RESULTS

The criteria presented in this report provide guidance in developing and tailoring specific design, functional, and performance requirements for the SHF terminal.

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INTRODUCTION

The Super-High Frequency (SHF) SATCOM Terminal is a shipboard communications system required to automatically acquire and track satellites in geosynchronous orbit while subject to ship motion. The requirements and constraints imposed by ship motion on the design of the terminal significantly impact several areas of its performance, including its ability to track satellites, correct for Doppler frequency shift, and maintain mechanical stiffness and structural integrity.

This study was initiated during development of the Phase-III SHF-Terminal Specification (reference 1) to ensure that all relevant characteristics related to ship motion are incorporated and appropriately specified. Its purpose is to define the realistic requirements imposed on the SHF SATCOM Terminal by ship motion under operational conditions. The requirements disclosed in this study provide guidance in developing and tailoring specific design, functional, and performance requirements for the SHF terminal.

Mathematically complex procedures are usually used to predict translational and rotational displacements, velocities, and accelerations experienced by an item of shipboard equipment as a result of ship motion. These are typically handled by a computer program, such as the Standard Ship-Motion Computer Program (SMP) (reference 2), capable of ship response and seakeeping analyses. This method of analysis requires detailed information about ship geometry, mass distribution, and equipment-mounting location. Much of this information is not readily available to activities not engaged in ship design. When, as in this case, installation is planned for a number of classes of ships and none of the mounting locations are specified, the workload is compounded. For this situation, a simpler method is needed for generating estimates of the worst-case motion and loads experienced by the equipment.

Installation of the SHF SATCOM Terminal is proposed for all Navy surface combatants. The variety of hull designs involved presents a wide spectrum of motion amplitudes and periods that must be accommodated by the terminal. When equipment is designed for installation aboard several types of ships, the magnitude of each design variable is typically specified to be equal to or greater than the worst-case value expected among all of the ships and installations being considered. This method of specifying equipment design parameters is typically used because it assures that the worst-case conditions expected are within its specified performance envelope, regardless of the type of ship on which it is mounted. A notable consequence of this method, however, does exist. Peak values for design variables from two or more different types of ships may inadvertently be applied simultaneously to computations for deriving performance or functional requirements. This coupling of unrelated variables may result in grossly overestimating the requirement, placing an unnecessary or unrealistic burden on the design. Take, for example, coupling the large-amplitude, short-period roll and pitch of a small ship, such as a frigate, with the dimensions of a large ship, such as an aircraft carrier. The computed load factors can be more than double those that may be realistically expected aboard any ship in the given sea condition. This report presents an alternative method of determining representative ship motion; i.e., it avoids this failing and yields realistic peak values.

SHIP-MOTION AND PERFORMANCE PARAMETERS

The motion experienced by a ship is typically described in terms of six components, each corresponding to one of the ship's 6 degrees-of-freedom. The conventional parameters used to describe them are defined in table 1.

Table 1. Ship-motion parameters.

Parameter	Definition
Roll	Oscillatory motion of a ship about the longitudinal (x) axis.
Pitch	Oscillatory motion of a ship about the transverse (y) axis.
Yaw	Oscillatory motion of a ship about the vertical (z) axis.
Surge	Fore and aft motion of a ship along the longitudinal (x) axis.
Sway	Lateral motion of a ship along the transverse (y) axis.
Heave	Up and down motion of a ship along the vertical (z) axis.

For estimating purposes, each of the six components of ship motion can be assumed to be oscillatory, characterized by a peak amplitude and modal period as listed in table 2.

Table 2. Ship-motion characteristics.

Parameter	Amplitude		Period	
	Maximum amplitude	Symbol	Period	Symbol
Roll	Maximum roll amplitude	ϕ	roll period	T_r
Pitch	Maximum pitch amplitude	θ	pitch period	T_p
Yaw	Maximum yaw amplitude	μ	yaw period	T_μ
Surge	Maximum surge amplitude	δ_x	surge period	T_x
Sway	Maximum sway amplitude	δ_y	sway period	T_y
Heave	Maximum heave amplitude	δ_z	heave period	T_z

Defining the terminal requirements imposed by ship motion first requires identifying measurable engineering features of the ship-motion interface that characterize them. These features are called performance characteristics. Computational methods are then developed for accurately estimating worst-case performance-characteristic values for each terminal installation. Finally, procedures are developed for establishing the realistic worst-case values of these performance characteristics in multiple-ship terminal installations. These values are used to (1) verify that performance requirements for the terminal are appropriately specified or (2) establish new requirements.

The following facets of the SHF Terminal's function and performance are influenced by ship motion and are primary considerations in developing the design specification. They are (1) its mechanical design and structural integrity, (2) its ability to acquire and track satellites, and (3) its ability to correct for Doppler frequency shift. Characteristics that describe the ship-motion interface and also serve to quantify performance requirements in these functional areas include maximum rectilinear velocity, acceleration, and jerk, and maximum angular velocity and acceleration. Table 3 summarizes the parameters that are critical to terminal performance and the performance characteristics that impose constraints on each of these parameters.

Table 3. Performance characteristics.

Functional Parameters	Performance Characteristics
Mechanical design and structural integrity	Maximum rectilinear acceleration
Acquiring and tracking satellites	Maximum angular velocity Maximum angular acceleration
Doppler frequency shift correction	Maximum rectilinear velocity Maximum rectilinear acceleration Maximum rectilinear jerk

The rectilinear performance characteristics result from the three linear-motion components (surge, sway, and heave) and translation due to the three angular-motion components (roll, pitch, and yaw) for specific shipboard mounting locations. The rectilinear components of ship motion increase with distance from the ship's axes of motion. Thus, the net rectilinear effects experienced by a piece of equipment depend upon its location aboard the ship. The relevant location parameters are shown in table 4.

Table 4. Mounting-location parameters.

Longitudinal distance forward or aft of the cg	X
Transverse distance port or starboard of the cg	Y
Vertical distance above the cg	Z

DEVELOPMENT OF TERMINAL-PERFORMANCE CHARACTERISTICS

APPROACH

A systematic approach is used to develop the terminal's performance requirements. The performance characteristics are computed for (1) each type ship, (2) the range of mounting locations on each platform, and (3) each sea condition. The value of each performance characteristic imposing the most stringent design limitations is then extracted from its group. These worst-case values for each performance characteristic are compiled into a demarcation set. This set represents a realistic performance-requirements envelope for all ships and installation configurations. The steps to this systematic approach—used to compile, manipulate, and reduce the ship-motion data—are outlined here and explained later in greater detail. The procedures taken in this approach are depicted as a flow diagram in figure 1.

1. **Ship List.** The ship classes on which SHF terminals are to be mounted are identified in the equipment specification. A manageable number of ships are selected to make up a representative list.
2. **Data Sources.** Sources are identified to provide dimensions, stability characteristics, operational requirements, and mounting location data for each ship on the representative list.
3. **Ship Dimensions.** The significant dimensions of each class are determined.
4. **Stability Characteristics.** The stability characteristics of each ship type are obtained.
5. **Operational/Survival Sea-State Conditions.** For each class of ship and each significant operating condition, the worst-case sea-state environment is determined.
6. **Ship-Motion Characteristics.** The dimensional, stability, and operational data are used in conjunction with DoD standards defining ship motion and attitude to predict worst-case ship-motion characteristics for each ship installation.
7. **Antenna-Mounting Locations.** A shipboard-mounting envelope is defined for the terminal on each class of ship.
8. **Computational Methods.** Computational methods are developed to determine the performance characteristics imposed on a terminal, given its mounting location and the ship's motion characteristics.
9. **Performance Characteristics.** For each specific operating condition, the worst-case value of each performance characteristic is determined from the data for all the ships and mounting arrangements. The set of worst-case values represents the performance-requirements envelope for that operating condition and is used as guidance in developing the equipment specification.

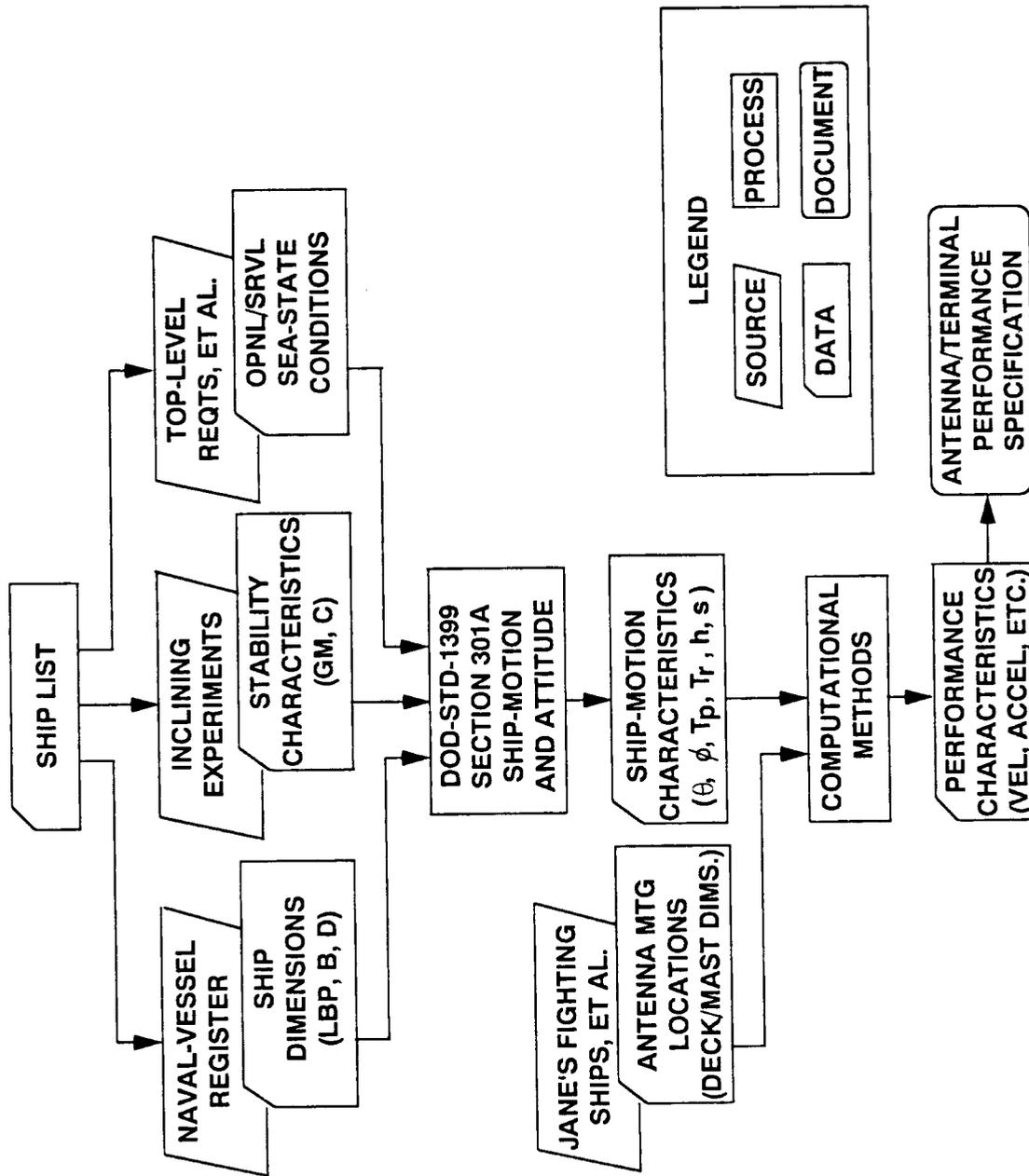


Figure 1. Procedural diagram for developing terminal-performance characteristics.

SHIP LIST

The SHF SATCOM Terminal is planned to be installed on all surface combatant ships. In addition, several amphibious warfare, underway replenishment, material support, and auxiliary force ships have been targeted for installation. Table 5 lists the ship types comprising the current list of candidate vessels.

Table 5. Ship types selected for terminal installation.

Aircraft carriers	CVN, CV
Cruisers	CGN, CG
Destroyers	DDG, DD
Frigates	FFG, FF
Amphibious warships	LCC, LHA, LHD, LPD
Command ships	AGF
Combat logistic ships	AOE, AOR, AD
SURTASS	T-AGOS

This list of ship types represents approximately 330 vessels in 38 classes. Collecting and processing the data needed for analyzing every ship is felt to be unnecessary, particularly when one considers the large number of duplications. A representative list that includes ships from each of the major classes and all apparent outliers should be adequate for this study. The ships chosen to make up this list are shown in table 6.

Table 6. Representative ships list.

CVN 68	Nimitz
CV 59	Forrestal
CGN 38	Virginia
CGN 36	California
CGN 9	Long Beach
CG 47	Ticonderoga
CG 26	Belknap
DDG 51	Arleigh Burke
DDG 993	Kidd
DD 963	Spruance
FFG 7	Oliver Hazard Perry
FF 1052	Knox
LCC 19	Blue Ridge
LHA 1	Tarawa
LHD 1	Wasp
LPD 1	Raleigh
AGF 11	Coronado
AOE 1	Sacramento
AOR 1	Wichita
AD 41	Yellowstone
T-AGOS 1	Stalwart

The analysis methods used in this study apply only to conventional, single-hull surface ships. An analysis of the motion of small-waterplane-area twin-hull (SWATH) ships, that include the T-AGOS 19 and T-AGOS 23 classes, is beyond the scope of this paper.

DATA SOURCES

For each of the ships on the representative list, the data needed to compute performance-characteristics values were gathered. The types and sources of this data are listed in table 7.

Table 7. Data sources.

Ship Dimensions	Naval Vessel Register (ref 3)
Stability Characteristics	Inclining Experiment Reports NAVSEA — Stability Division
Operational/Survival Sea-State Conditions	Top Level Requirements (ref 4-6) NAVSEA — Hydrodynamics Perf Div David Taylor Research Center
Antenna-Mounting Locations	Jane's Fighting Ships (ref 7) NAVSEA — Arrangements Div

SHIP DIMENSIONS

To a certain extent, the general hull shapes of most conventional naval surface ships are geometrically similar. The main differences between them are their scale and their proportions between length and beam. Considering this assumption to be valid for estimating purposes, it may then be argued that a hull shape's dynamic response to a given sea state can be characterized by two dimensions: its length between perpendiculars (LBP) and its waterline beam (B). Another significant ship dimension is its draft. It provides a reference indicator of the ship's displacement and loading condition. These dimensions are available for all surface combatants in the Naval Vessel Register (reference 3). Dimensional data used in this study may be found in Appendix A.

STABILITY CHARACTERISTICS

In a simplified sense, when inclined, a ship's stability is its tendency to restore itself to the upright position. The mechanics of how a righting moment is generated is illustrated in figure 2. When a ship is inclined, its center of buoyancy becomes relocated at the new center of displaced volume. A righting moment is created by the force of the displacement downward through the center of gravity and an equal force upward through the center of buoyancy. This righting moment is proportional to the displacement, to the sign of the angle of inclination, and to the metacentric height (GM). The metacentric height, then, is a useful indicator of a ship's initial stability. A ship's metacentric height, for a given load condition, may be measured by the inclining experiment and recomputed for other load conditions.

b = CENTER OF BUOYANCY
 G = CENTER OF GRAVITY
 M = METACENTER
 \overline{GM} = METACENTRIC HEIGHT
 W = DISPLACEMENT
 ϕ = ANGLE OF HEEL
 B = BEAM
 C = ROLL CONSTANT

$$\text{RIGHTING MOMENT} = W \cdot \overline{GM} \cdot \sin \phi$$

$$\text{ROLL PERIOD} = \frac{B \cdot C}{\sqrt{GM}}$$

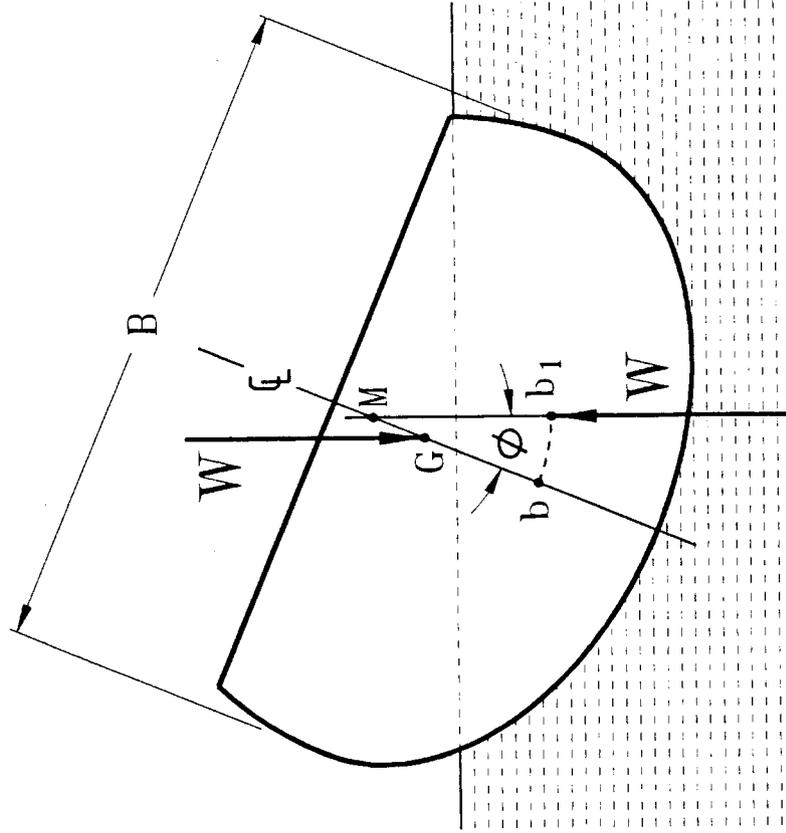


Figure 2. Initial-stability parameters.

A ship's roll period is inversely proportional to the square root of the metacentric height and proportional to the beam and the roll constant. The roll constant is an indicator of a ship's rolling inertia and is also determined during the inclining experiment. Values for these variables and other inclining experiment data for naval ships are maintained by NAVSEA, Stability Division. Stability data used in this study may be found in Appendix A.

OPERATIONAL/SURVIVAL SEA-STATE CONDITIONS

Performance of the SHF terminal in extreme operational or survival conditions must be satisfactory and consistent with the mission of the ship on which it is installed. Three operating conditions for surface ships, applicable to specific sea-state ranges, are listed in table 8. Descriptions of ships' performance requirements for each condition, taken from Top Level Requirements (TLR) (references 4-6) are also provided. These three conditions appear to suitably describe the operating conditions and requirements of the SHF terminal and are so adopted within this report.

Table 8. Operating conditions.

Condition	Description
Continuous	Continous efficient operation (other than replenishment). Ship must be capable of maintaining course and speed to suit mission requirements.
Limited	Limited operation and capability of continuing its mission without returning to port for repairs after sea subsides. Ship must be capable of continuing mission (60% of the time) while maintaining course and speed from the standpoint of ship motion.
Survival	Survivability without serious damage to mission-essential equipment.

The range of sea states for which each of these operating conditions applies to a ship is typically provided in the Top Level Requirements for its class. For many of the older surface combatants, however, no TLRs were found to exist. Approved Characteristics or Required Operational Capabilities (ROC) are the equivalent documents used in the development of many older ships. However, references to sea-state condition were found to be typically avoided in these older documents. The reason for this is that more than 20 different scales were in common use for sea-state conditions. In 1983, however, a standardized scale was adopted by NATO countries in STANAG No. 4194 (reference 8). (See table 9.) Sea spectra applied to analysis are those defined by the Brethschneider formulation (reference 9). To fill the void where sea-state numbers had not been assigned to operating conditions, assistance was requested from the various NAVSEA type desks in interpreting Approved Characteristics and ROCs. Guidance was also requested from the NAVSEA Hydrodynamics Performance Division and from the David Taylor Research Center. The general consensus from these inquiries was that sea states of 6, 7, and 8 may be assumed to apply respectively to the three stated operating condition descriptions in increasing order of severity.

Table 9. NATO sea-state numeral table.

Sea State No.	Significant Wave Height (m)		Modal Wave Period (sec)	
	Range	Mean	Range	Most Probable
0-1	0 - 0.1	0.05	-	-
2	0.1 - 0.5	0.3	3.3 - 12.8	7.5
3	0.5 - 1.25	0.88	5.0 - 14.8	7.5
4	1.25 - 2.5	1.88	6.1 - 15.2	8.8
5	2.5 - 4	3.25	8.3 - 15.5	9.7
6	4 - 6	5	9.8 - 16.2	12.4
7	6 - 9	7.5	11.8 - 18.5	15.0
8	9 - 14	11.5	14.2 - 18.6	16.4
>8	>14	>14	15.7 - 23.7	20.0

SHIP-MOTION CHARACTERISTICS

The implement used to quantify each ship's motion characteristics is DoD-STD-1399, Section 301A: Interface Standard for Shipboard Systems—Ship Motion and Attitude (reference 10). This document was developed to simplify estimating ship-motion loading factors on shipboard foundations. It provides (1) tables that list peak amplitudes and modal periods for roll and pitch motion and (2) peak values for surge and heave acceleration. Yaw and sway are disregarded since the forces they generate normally have a lesser magnitude. The ship's dimensions and stability characteristics and the operational sea-state information previously discussed comprise the information needed to use the tables. Table 10 summarizes the input data required by the tables and the information provided by them. These tables are reproduced as look-up tables in Appendix A; the values they provide are based on empirical data.

Table 10. Ship-motion tables—input/output data.

Input Data		Output Data	
Sea state	SS	Max roll angle	ϕ
Length btwn perpendiculars	LBP	Roll period	T_r
Waterline beam	B	Max pitch angle	θ
Metacentric height	GM	Pitch period	T_p
Roll constant	C	Heave acceleration	h
		Surge acceleration	s

ANTENNA-MOUNTING LOCATIONS

The motions experienced by items of shipboard equipment, and the resulting forces acting on them, vary, depending upon their location on the ship. These dynamic effects increase with distance from the ship's motion axes. The location of a piece of equipment is therefore critical to computing its performance parameters. For this study, the motion axes of each ship are assumed to pass through its center of gravity.

The mounting location of the SHF terminal aboard any specific ship has not yet been designated. Any location where mounting is possible, even on towers yet to be constructed, must be considered for installing the terminal. This is because of a possible requirement for a dual antenna with hand-over capability and the need to avoid blanketing the antenna with the superstructure. Until specific mounting locations are designated, an envelope must be estimated in which the terminal is sure to be mounted. In this study, the configuration of the mounting envelope is assumed to be the equivalent of a rectangular box with dimensions equal to the overall length of the ship, the maximum beam, and the height of the highest mast platform. A typical mounting envelope is illustrated in figure 3. Attempts made to reduce the size of this envelope or to change its shape yielded reductions in the magnitudes of performance requirements of less than 35 percent. In general, this reduction is not felt to be sufficient to warrant the additional effort required. This method of approximating the mounting envelope most assuredly encircles all possible mounting locations, simplifies comparisons between installations on different ship classes, and provides conservative, but realistic, worst-case estimates. Upon reviewing the performance characteristics for the 21 ships evaluated, one or two outlier ships were found to be unnecessarily expanding the requirements envelope for the Fleet. For these isolated cases, we determined it reasonable to substitute a more realistic approach to defining the mounting envelope and, if necessary, to place limits on the system's mounting location on these ships. Decisions of this type were limited to the two worst-case ships for any single operating condition. Details regarding the vessels targeted and the methodologies used are not included in this report. Geometrical dimensions used to estimate mounting envelope sizes are obtained from NAVSEA, Ship Arrangements Division, and from Jane's Fighting Ships (reference 7). Mounting envelope dimensions used in this study are listed in Appendix A.

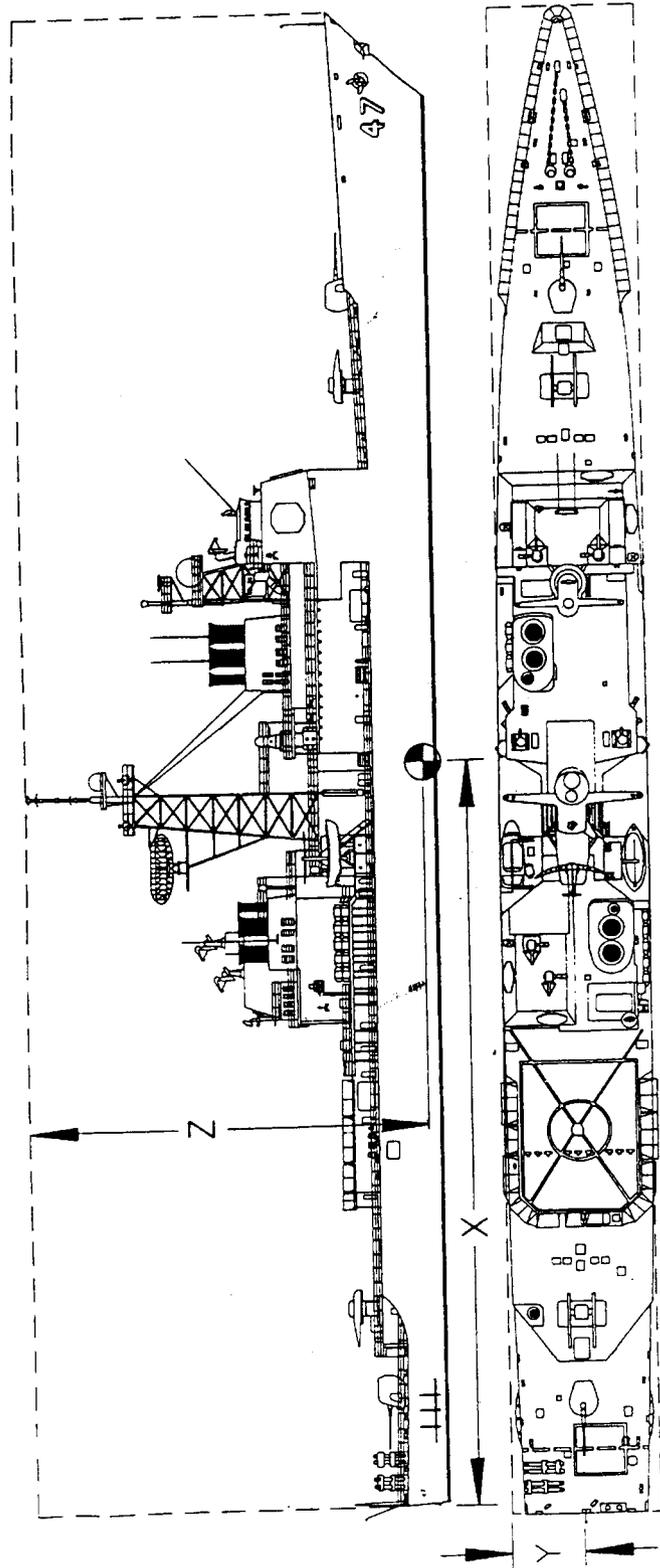


Figure 3. Mounting envelope.

COMPUTATIONAL METHODS

Equations were developed to characterize, with reasonable accuracy, the expected dynamic environment of a point on a ship in moderate seas, but without using seakeeping analysis techniques or ship modeling methods. The approach taken parallels one suggested by Sandberg (reference 11), who proposes a simple method for accurately estimating the dynamic forces at a point on a ship in a rough sea. He derived the equations of motion by applying Newton's second law at the point and further simplified them with well thought out approximations and assumptions. His approach offers a simple, yet arguably accurate method for obtaining conservative acceleration and force estimates. The equations developed for this study merely expand on Sandberg's approach to include not only acceleration and force, but also displacements, velocities, and jerks.

In addition to providing tables of ship-motion characteristics values, DoD-STD-1399, Section 301A (reference 6), also provides loading-factor equations for estimating the forces experienced by shipboard foundations due to ship motion. These equations use only the tabulated data and the foundation-location data to arrive at the loading-factor values. The equations used in DoD-STD-1399, Section 301A, are consistent with those developed by Sandberg. However, Section 301A goes a step further in solving the equations by supplying values for the input variables, listed in tables as functions of the ship dimensions and the operating condition. As in Sandberg's method, Section 301A falls short of detailing methods for estimating other ship-motion effects.

The equations used in this study are developed following the approach used by Sandberg, but expanded to describe peak values for the performance characteristics. Their development seeks to describe the peak angular and rectilinear-motion amplitudes, velocities, accelerations, and jerks experienced at a point on a ship. These descriptions are expressed in terms of the peak amplitudes and modal periods of pitch, roll, surge, and heave, and the location of the point. Loading-factor equations are also developed to assure that these equations are consistent with those of Section 301A. Having achieved conformity, the next obvious, but presumptive step, is to apply the numerical values for ship-motion characteristics provided in Section 301A to the performance-characteristic equations. The result is a reasonably accurate, but conservative estimate of the terminal's performance characteristics. Development of these equations is described in Appendix B.

A systematic approach for estimating ship motion and deriving performance characteristics has been proposed. Data sources have been identified and the required computational methods have been developed. The key to making this approach work is to simplify storing and manipulating data. By the nature of this task, a worksheet, or "spreadsheet," appears to be the ideal platform on which to build the model. Worksheets provide the tools needed for data entry, storage, manipulation, calculation, analysis, and reporting. They also typically include many features designed to allow easy expansion, editing, the use of look-up tables, projections, decision-making, graphical presentation, and transfer of data to other programs.

The worksheet used in this effort is Microsoft Excel (reference 12). A sample printout is included as Appendix A. The worksheet is set up to show only one sea state or one operating condition at a time. This example shows a worksheet for the sea-state 6 condition. The first page (A-3) of the worksheet accommodates data entry. Each ship's dimensions, stability characteristics, mounting-envelope dimensions, and sea-state conditions are input. The ship-motion characteristics, listed on page A-4, are determined from the input data and look-up tables. The next

three pages list the angular velocities and accelerations computed for each ship, followed by linear velocities, accelerations, jerks, loading factors, and velocities, with top ships speed worked in. The worst-case performance-characteristics values are summarized on page A-8, and the look-up tables of ship-motion characteristics are shown on page A-9.

The significance of the performance-characteristics values is difficult to absorb from a table of values alone. A graphic depiction of these values provides a more palatable means of understanding the relationship between these numbers. The method of presentation used in this study is a plot of the performance-characteristics values on a graph comparing peak velocity against period. This representation of performance characteristics for the CG-47 in sea-state 6 is shown in figure 4. Angular and rectilinear components of motion are presented separately for clarity. On such a graph, constant velocity is represented as a horizontal line; constant acceleration is a line of constant slope; constant jerk is a parabolic curve; and constant displacement is inversely proportional to the period. The plot of performance characteristics as constants establishes the boundaries to a zone, called the motion envelope, within which all ship motion takes place. This type of plot, called conformal mapping, provides a graphic representation of the range and types of motion expected. Figure 5 shows the motion envelopes for the same ship in sea-state 6, 7, and 8 conditions.

By superimposing the motion envelopes for antennas mounted on each of the ships on the representative list in a specific sea state or operating condition, the overall motion requirements for the antenna in that condition are developed. Figure 6 shows the superimposed angular-motion and rectilinear-motion envelopes of all the ships in the condition of continuous efficient operation. Figure 7 shows the motion envelopes for the terminal, developed in this way, for the continuous, limited, and survival conditions.

PERFORMANCE CHARACTERISTICS

The worst-case performance characteristics determined in this study are summarized in table 11. These represent the most severe requirements imposed on the terminal by the ship-motion interface. Values are listed for the three significant operating conditions. These performance characteristics impose additional constraints on various facets of the terminal's design, including its structural integrity, its ability to acquire and track satellites, and its ability to correct for Doppler frequency shift.

Table 11. Maximum performance-characteristics values.

Performance Characteristics	Continuous Efficient Operation	Limited Operation	Survival Condition
Max angular velocity	12.0 deg/sec	23.0 deg/sec	26.6 deg/sec
Max angular acceleration	7.8 deg/sec ²	13.7 deg/sec ²	17.3 deg/sec ²
Max rectilinear velocity	40.8 ft/sec	57.8 ft/sec	90.5 ft/sec
Max rectilinear acceleration	33.2 ft/sec ²	55.5 ft/sec ²	89.3 ft/sec ²
Max rectilinear jerk	35.6 ft/sec ³	64.8 ft/sec ³	120.0 ft/sec ³

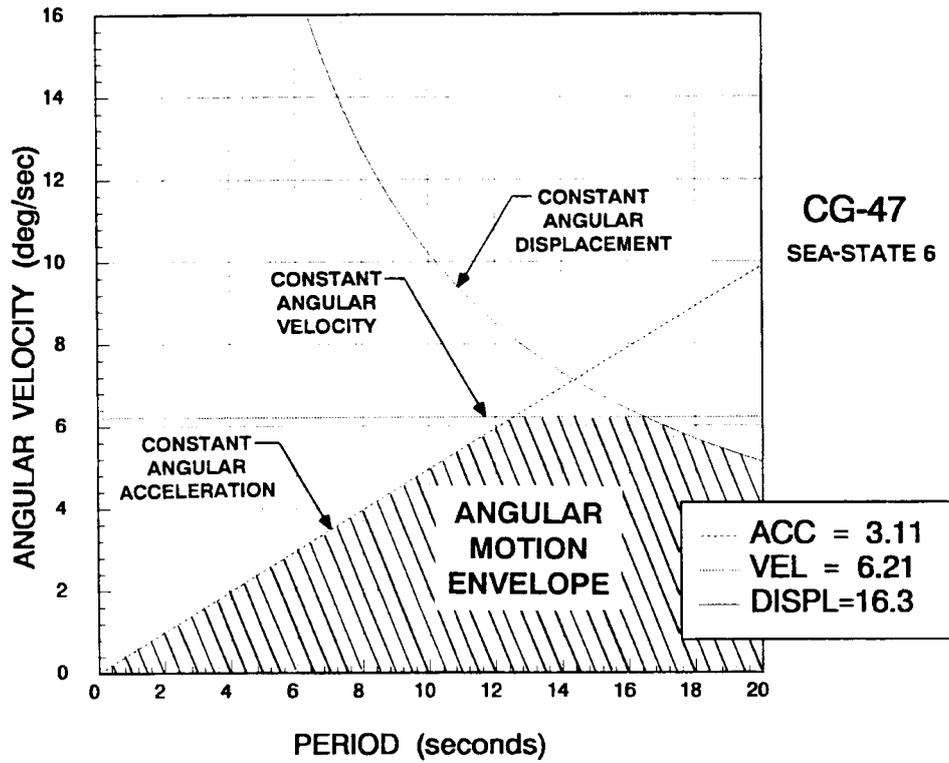


Figure 4a. Angular-motion envelope depiction.

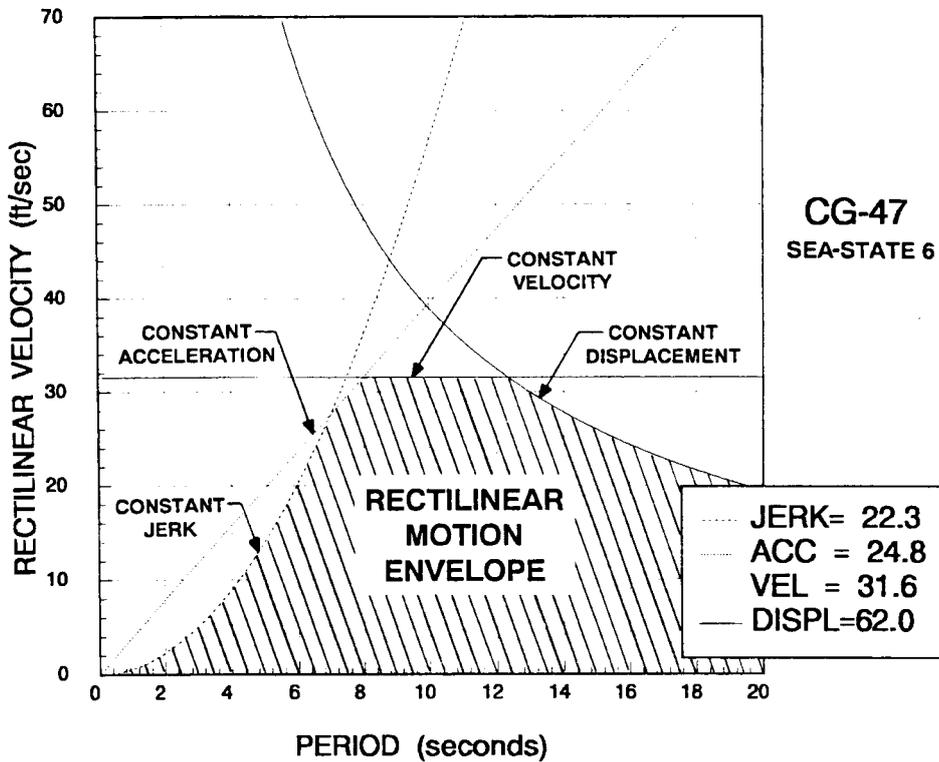


Figure 4b. Rectilinear-motion envelope depiction.

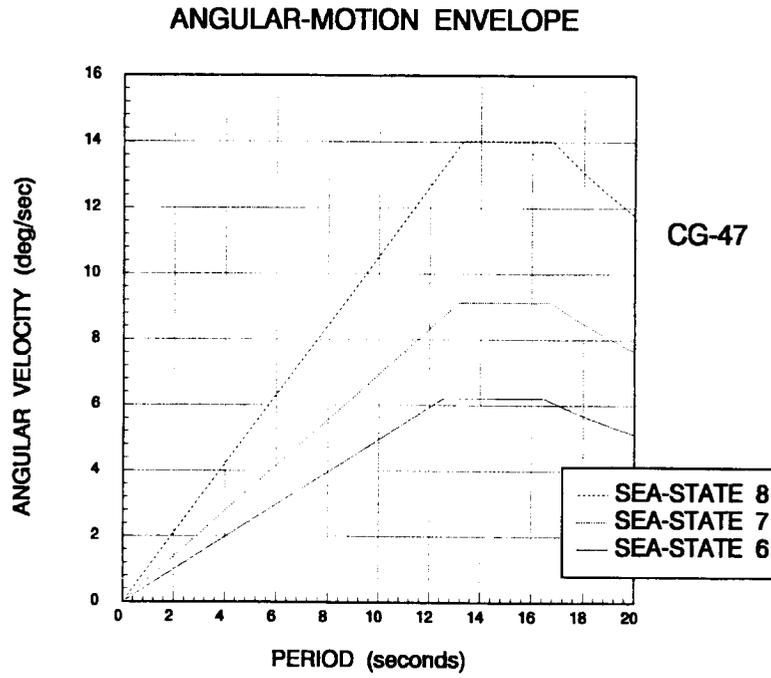


Figure 5a. Angular-motion envelopes for CG-47.

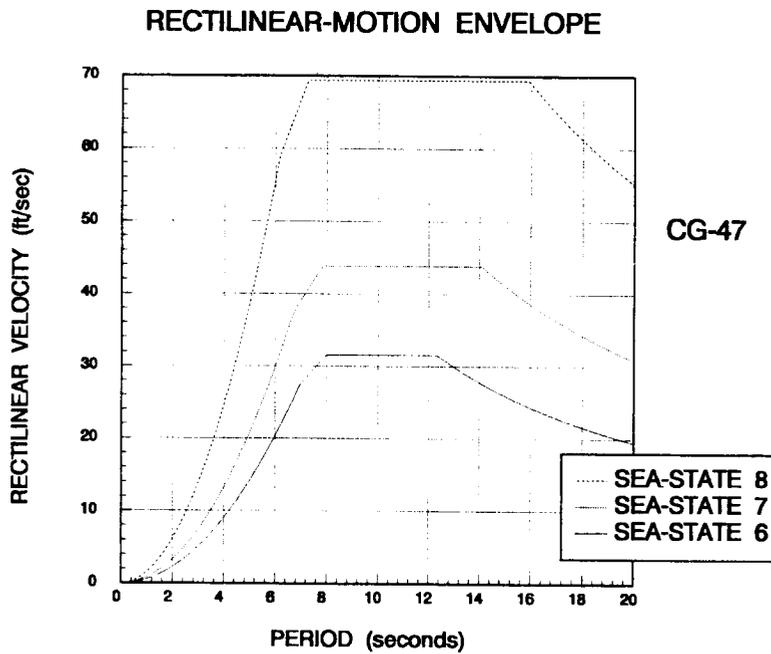


Figure 5b. Rectilinear-motion envelopes for CG-47.

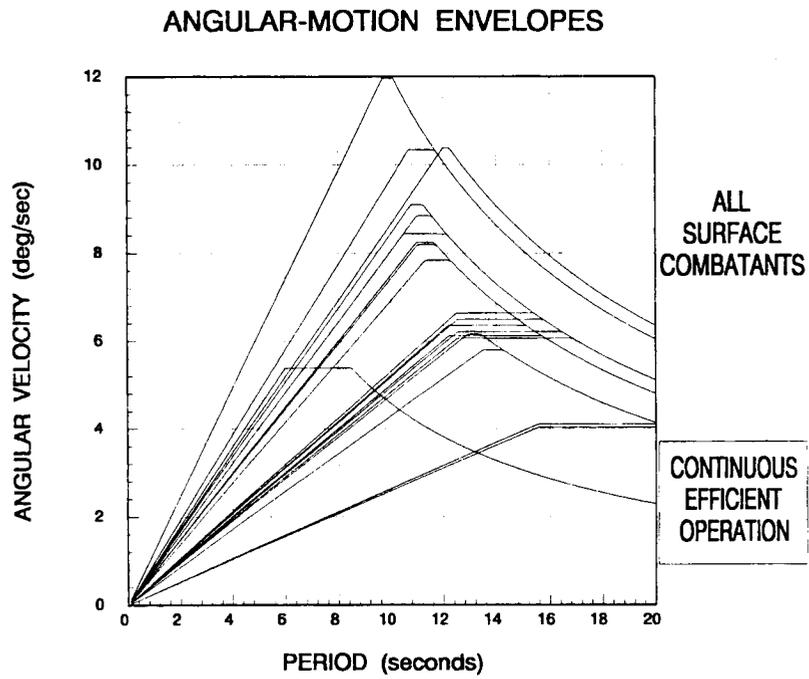


Figure 6a. Angular-motion envelopes for surface combatants.

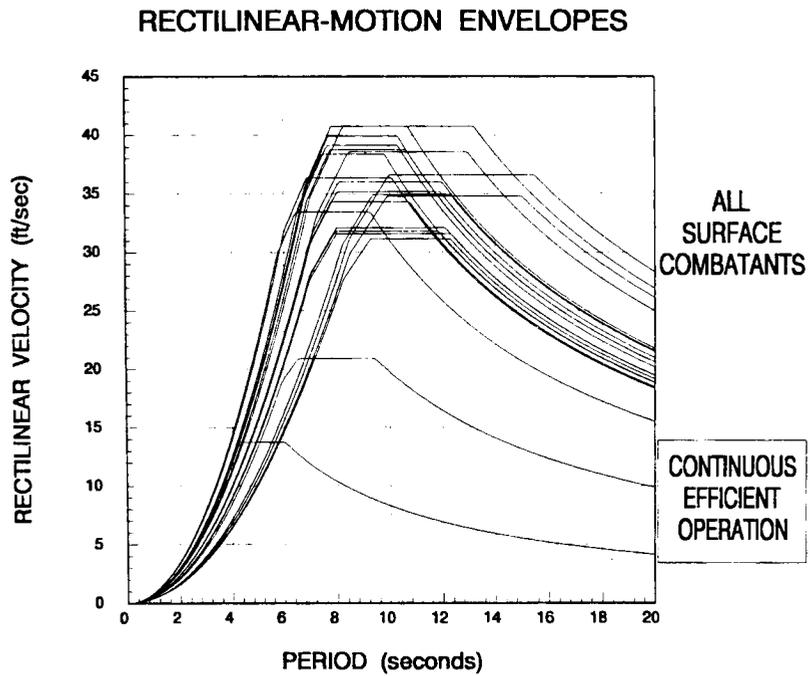


Figure 6b. Rectilinear-motion envelopes for surface combatants.

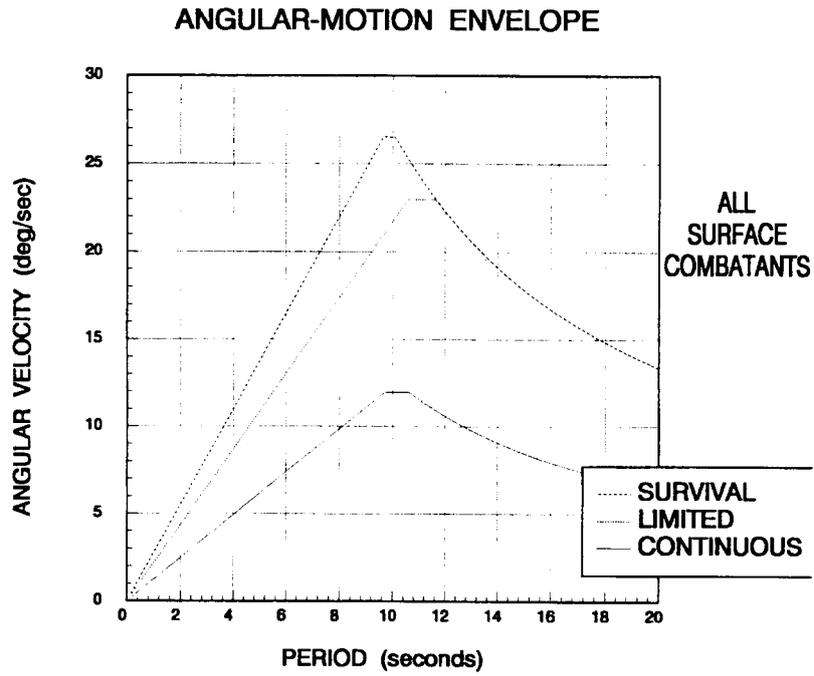


Figure 7a. Angular-motion envelopes for all surface combatants.

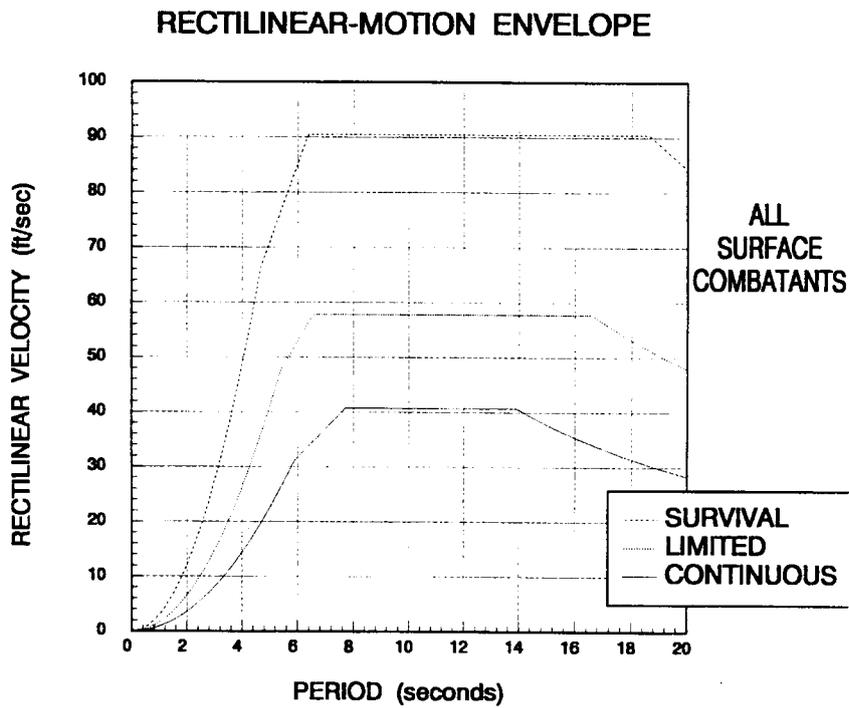


Figure 7b. Rectilinear-motion envelopes for all surface combatants.

CONCLUSION AND RECOMMENDATIONS

APPLICATION

Specific motion-related design requirements may be developed using as guidance the values in table 11 for the performance characteristics and their components as described in Appendix A. Loading factors, used in computing structural loads, are determined following DoD-STD-1399, Section 301A (reference 10), from the directional components of the rectilinear acceleration with gravity components added. Loading-factor values are listed in table 12.

Tracking capability required of the antenna-pointing system must allow for angular velocity and angular acceleration of the equipment platform. Tracking requirements are determined directly from the angular-motion performance characteristics and their directional components. Their values are listed in table 13.

Table 12. Loading-factor values.

Loading Factors		Continuous Efficient Operation	Limited Operation	Survival Condition
Longitudinal LF	A_x	14.4 ft/sec ²	30.5 ft/sec ²	33.2 ft/sec ²
Transverse LF	A_y	34.3 ft/sec ²	62.0 ft/sec ²	66.0 ft/sec ²
Vertical LF	A_z	57.1 ft/sec ³	83.2 ft/sec ²	87.1 ft/sec ²

Table 13. Satellite-tracking requirements.

Tracking Motion	Continuous Efficient Operation	Limited Operation
Maximum Roll Velocity	11.5 deg/sec	21.8 deg/sec
Maximum Pitch Velocity	3.1 deg/sec	7.3 deg/sec
Maximum Yaw Velocity	1.6 deg/sec	3.7 deg/sec
Combined Angular Velocity	12.0 deg/sec	23.0 deg/sec
Maximum Roll Acceleration	7.0 deg/sec ²	11.3 deg/sec ²
Maximum Pitch Acceleration	4.9 deg/sec ²	7.7 deg/sec ²
Maximum Yaw Acceleration	2.5 deg/sec ²	3.8 deg/sec ²
Combined Angular Acceleration	7.8 deg/sec ²	13.7 deg/sec ²

Doppler frequency shift and its derivatives are directly proportional to the rectilinear velocity and its derivatives. The signal-processing requirements of the terminal may be computed directly from the performance-characteristics values. The tracking requirements, based on a carrier frequency of 8 GHz, are listed in table 14.

Table 14. Signal-processing requirements.

Tracking Motion	Continuous Efficient Operation	Limited Operation
Doppler Shift Doppler Shift Rate Doppler Shift Rate Change	331 Hz 270 Hz/sec 290 Hz/sec ²	454 Hz 451 Hz/sec 527 Hz/sec ²

GLOSSARY

Amplitude. The amplitude of an oscillatory record is the difference between the mean value and individual peak (or trough).

Beam. The beam (B) is the breadth of a ship's hull.

Center of buoyancy. The center of buoyancy (b) is the geometric center of the immersed volume of a ship's hull. The sum of all buoyancy forces acting on the hull is equal in magnitude to the ship's displacement but acts vertically with a line of action passing through the center of buoyancy.

Center of gravity. The center of gravity (G) is the locus of all gravitational forces of the entire ship. It is the point about which the moment of all weights in the ship is equal to zero.

Displacement. The displacement (W) of a ship is the weight of the seawater it displaces when floating freely. Archimede's Principle holds that the displacement is equal to the weight of the ship and its cargo.

Draft. The draft (D) is the vertical height of the waterline above the baseline. It is measured using draft scales located near the forward and aft perpendiculars. Draft measurements indicate the displacement and loading condition of the ship.

Heave. Heave is the up and down motion of a ship along the vertical (Z) axis.

Length between perpendiculars. The length between perpendiculars (LBP) is the distance on a ship from the forward perpendicular to its aft perpendicular; the length of its hull at the waterline.

Loading factor. The loading factor is a calculated number given in terms of gravitational and dynamic acceleration which, when multiplied by the mass of a structure and divided by the gravitational constant (32.2 ft/sec^2), determines the design load as a result of ship motion and gravity.

Metacenter. The metacenter (M) is the point of intersection of two successive lines of action of the buoyancy force as a ship is inclined through a small angle.

Metacentric height. The metacentric height is the distance GM between the center of gravity and the metacenter of a ship. It is a measure of how much a ship's center of buoyancy moves when it is heeled and a measure of its initial stability or "stiffness."

Modal wave period. In a sea spectra, the modal wave period is the period of maximum wave energy and defines the peak location of the spectrum.

Performance characteristics. Performance characteristics are measurable features of the ship-motion interface that describe the shipboard environment.

Pitch. Pitch is the oscillatory motion of a ship about the transverse (Y) axis.

Roll. Roll is the oscillatory motion of a ship about the longitudinal (X) axis.

Roll constant. The roll constant (C) is an indication of a ship's rolling inertia, based on experimental results, used to predict its roll period under various load conditions. Values range from $0.38\text{--}0.50 \text{ sec-ft}^{-1/2}$.

Sea State. Sea state (SS) is a term describing roughness of the sea condition. Sea conditions include wave height, period, and energy distribution. Sea states are determined according to STANAG 4154 and STANAG 4194.

Ship-motion characteristics. Ship-motion characteristics are measurable features, such as peak amplitude and modal period, that describe the dynamic motion due to roll, pitch, yaw, surge, sway, and heave.

Significant wave height. If all the wave heights of a wave record are measured, the significant wave height is the mean value of the highest one-third of them.

Stability. Stability is a term indicating the tendency of a ship, when inclined, to restore itself to the upright position.

Surge. Surge is the fore and aft motion of a ship along the longitudinal (X) axis.

Sway. Sway is the lateral motion of a ship along the transverse (Y) axis.

Yaw. Yaw is the oscillatory motion of a ship about the vertical (Z) axis.

REFERENCES

1. Space and Naval Warfare Systems Command. 1992. "Specification for SHF Satellite Communications Terminal, AN/WSC-6(V) XX." Draft (5 May).
2. Meyers, W. G. Applebee, T. R., and Baitis, A. E. 1981. "Users Manual for the Standard Ship-Motion Program, SMP," David W. Taylor Naval Ship Research and Development Center, DTNSRDC/SPD-0936-1, Bethesda, MD, September 1981.
3. Naval Vessel Register—Ships Data Book, NAVSEA S0300-A4-MAN-A10/(U) (U), 0910-LP-447-5800, vol. I, 1 July 1991.
4. DDG 47 Guideship, SAIP Project No. 266.77, Top Level Requirements (TLR); Promulgation of (U), OPNAVINST C9010.314 Ser 37/C702990, 28 Sep 1975.
5. Top Level Requirements (TLR) for Arleigh Burke Class Guided Missile Destroyer (DDG 51) (U), OPNAVINST C9010. Ser 03C/5C391543, 19 August 1985.
6. Promulgation of Revision to Auxiliary Ocean Surveillance Ship (T-AGOS), Top Level Requirements (TLR) (U), OPNAVINST C9010.315 B, 9 October 1984.
7. *Jane's Fighting Ships*, 1992/93, ed. Capt. R. Sharpe, Jane's Information Group, Alexandria, VA, 1992.
8. Military Agency for Standardization (MAS). NATO, 1983. "Standardized Wave and Wind Environments and Shipboard Reporting of Sea Conditions." STANAG No. 4194, 6 April 1983.
9. Bretschneider, C. L. 1959. "Wave Variability and Spectra for Wind-Generated Gravity Waves." Department of the Army, Corps of Engineers Technical Memorandum 118.
10. DoD-STD-1399, Section 301A, Interface Standard for Shipboard Systems—Ship Motion and Attitude, 21 July 1986.
11. Sandberg, W. 1979. "The Estimation of Ship Motion-Induced Forces," *The Society of Naval Architects and Marine Engineers, STAR Symposium*, paper no. 21, Houston, Texas, April 25-28, 1979.
12. Microsoft Corporation. 1991. "Microsoft Excel Users Guide," version 3.0.

APPENDIX A

DATABASE WORKSHEET

SEA-STATE 6

INPUT DATA

OPERATING CONDITION:

Sea State 6

VESSEL	DIMENSIONAL DATA				INCLINING DATA				MOUNTING ENVELOPE			
	Displ (full load) W (tons)	Length Btwn Perp LBP (ft)	Waterline Beam B (ft)	Draft (full load) D (ft)	Metacntrc Height GM (ft)	Roll Const C	Inclin Roll Period (sec)	Long Dist fwd of c.g. X (ft)	Trnsv Dist port of c.g. Y (ft)	Vert Dist abv c.g. Z (ft)		
T-AGOS 1	2,262	204	43.00	14.92	3.81	0.456		112.0	21.5	68		
FF 1052	4,291	415	46.60	15.65	4.11	0.450		219.0	23.5	83		
FFG 7	4,050	408	46.96	15.93	2.91	0.440		124.0	23.5	40		
CG 26	8,400	524	54.00	20.00	4.40		10.50	273.5	27.5	134		
CGN 36	11,653	570	60.67	22.73	4.40	0.400		298.0	30.5	143		
CGN 38	10,658	560	61.75	23.00	4.50	0.410		293.0	31.5	162		
DDG 51	8,276	466	59.00	20.27	4.87	0.480		252.0	33.0	128		
AGF 11	15,021	548	84.00	19.87	8.40		12.10	284.5	54.0	143		
AOR 1	39,406	640	96.00	34.61	10.74	0.416		329.5	48.0	140		
LPD 1	14,058	500	82.00	20.77	11.12		12.80	261.0	50.0	123		
LHD 1	40,702	778	106.00	26.75	13.45	0.430		200.0	55.0	127		
DD 963	9,275	529	55.00	21.85	2.54	0.480		281.5	27.5	133		
DDG 993	9,872	529	55.00	22.76	2.51	0.490		281.5	27.5	140		
LCC 19	18,982	580	82.00	26.28	5.32	0.460		310.0	54.0	151		
CG 47	10,144	529	55.00	23.17	2.44	0.510		283.5	27.5	140		
AOE 1	49,937	770	107.50	37.10	10.94	0.422		398.0	53.5	205		
AD 41	20,438	620	85.10	22.70	6.35		17.13	321.0	42.5	152		
CGN 9	17,110	690	73.00	24.32	4.01		18.50	360.5	36.0	207		
LHA 1	39,936	778	106.00	26.58	13.51	0.507		410.0	53.0	168		
CV 59	80,662	990	130.00	36.75	9.11	0.500		533.5	135.0	177		
CVN 68	94,937	1040	134.00	37.91	11.81		22.00	557.5	128.5	217		

OPERATIONAL DATA

SHIP MOTION CHARACTERISTICS

OPERATING CONDITION:

Sea State 6

Vessel	Ships Top Speed V (knots)	Sea State
T-AGOS 1	11	4
FF 1052	27	6
FFG 7	29	6
CG 26	33	6
CGN 36	31	6
CGN 38	31	6
DDG 51	32	6
AGF 11	21	6
AOR 1	20	6
LPD 1	22	6
LHD 1	23	6
DD 963	32	6
DDG 993	31	6
LCC 19	21	6
CG 47	31	6
AOE 1	28	6
AD 41	20	6
CGN 9	32	6
LHA 1	24	6
CV 59	33	6
CVN 68	33	6

Vessel	Roll Period Tr (sec)	Max Roll Angle (deg)	Pitch Period Tp (sec)	Max Pitch Angle (deg)	Heave Acctn h (g)	Surge Acctn s (g)
T-AGOS 1	10.04548	7	4	2	.10	.05
FF 1052	10.34374	19	6	3	.21	.10
FFG 7	12.11251	19	6	3	.21	.10
CG 26	10.50000	16	7	3	.16	.10
CGN 36	11.56932	16	7	3	.16	.10
CGN 38	11.93478	16	7	3	.16	.10
DDG 51	12.83302	16	6	3	.21	.10
AGF 11	12.10000	15	7	3	.16	.10
AOR 1	12.18604	15	7	3	.16	.10
LPD 1	12.80000	15	7	3	.16	.10
LHD 1	12.42834	13	8	2	.11	.05
DD 963	16.56483	16	7	3	.16	.10
DDG 993	17.01069	16	7	3	.16	.10
LCC 19	16.35369	15	7	3	.16	.10
CG 47	17.95717	16	7	3	.16	.10
AOE 1	13.71552	13	8	2	.11	.05
AD 41	17.13000	15	7	3	.16	.10
CGN 9	18.50000	16	7	3	.16	.10
LHA 1	14.62131	13	8	2	.11	.05
CV 59	21.53546	13	8	2	.11	.05
CVN 68	22.00000	13	8	2	.11	.05

* T-AGOS 1/SURTASS antenna continuous operation limited to sea state 4.

ANGULAR MOTION

OPERATING CONDITION:

Sea State 6

Vessel	Sea State	PEAK ANGULAR AMPLITUDE			PEAK ANGULAR VELOCITIES			PEAK ANG ACCELERATIONS		
		Max Roll Angle (deg)	Max Pitch Angle (deg)	Combined Angle (deg)	Roll Velocity (deg/sec)	Pitch Velocity (deg/sec)	Combined Ang Vel (deg/sec)	Roll Acctn (deg/s/s)	Pitch Acctn (deg/s/s)	Combined Ang Acctn (deg/s/s)
T-AGOS 1	4	7	2	7.28011	4.37832	3.14160	5.38881	2.73852	4.93480	5.64373
FF 1052	6	19	3	19.23538	11.54134	3.14160	11.96128	7.01065	3.28987	7.74418
FFG 7	6	19	3	19.23538	9.85597	3.14160	10.34456	5.11263	3.28987	6.07966
CG 26	6	16	3	16.27882	9.57438	2.69280	9.94585	5.72929	2.41704	6.21827
CGN 36	6	16	3	16.27882	8.68945	2.69280	9.09713	4.71915	2.41704	5.30212
CGN 38	6	16	3	16.27882	8.42336	2.69280	8.84332	4.43456	2.41704	5.05049
DDG 51	6	16	3	16.27882	7.83378	3.14160	8.44025	3.83650	3.28987	5.05315
AGF 11	6	15	3	15.29706	7.78908	2.69280	8.24141	4.04464	2.41704	4.71182
AOR 1	6	15	3	15.29706	7.73409	2.69280	8.18946	3.98773	2.41704	4.66306
LPD 1	6	15	3	15.29706	7.36311	2.69280	7.84006	3.61436	2.41704	4.34807
LHD 1	6	13	2	13.15295	6.57219	1.57080	6.75730	3.32259	1.23370	3.54424
DD 963	6	16	3	16.27882	6.06894	2.69280	6.63952	2.30200	2.41704	3.33786
DDG 993	6	16	3	16.27882	5.90987	2.69280	6.49444	2.18291	2.41704	3.25687
LCC 19	6	15	3	15.29706	5.76309	2.69280	6.36116	2.21421	2.41704	3.27793
CG 47	6	16	3	16.27882	5.59838	2.69280	6.21233	1.95886	2.41704	3.11115
AOE 1	6	13	2	13.15295	5.95540	1.57080	6.15908	2.72821	1.23370	2.99419
AD 41	6	15	3	15.29706	5.50192	2.69280	6.12554	2.01807	2.41704	3.14876
CGN 9	6	16	3	16.27882	5.43411	2.69280	6.06471	1.84559	2.41704	3.04111
LHA 1	6	13	2	13.15295	5.58647	1.57080	5.80311	2.40066	1.23370	2.69911
CV 59	6	13	2	13.15295	3.79288	1.57080	4.10528	1.10661	1.23370	1.65729
CVN 68	6	13	2	13.15295	3.71279	1.57080	4.03141	1.06037	1.23370	1.62678

RECTILINEAR MOTION

OPERATING CONDITION:

Sea State 6

Vessel	PEAK VELOCITIES				PEAK ACCELERATIONS				PEAK JERKS			
	Long Velocity (ft/sec)	Transv Velocity (ft/sec)	Vertical Velocity (ft/sec)	Combined Velocity (ft/sec)	Long Acctn (ft/s/s)	Transv Acctn (ft/s/s)	Vertical Acctn (ft/s/s)	Combined Acctn (ft/s/s)	Long Jerk (ft/s/s/s)	Transv Jerk (ft/s/s/s)	Vertical Jerk (ft/s/s/s)	Combined Jerk (ft/s/s/s)
T-AGOS 1	4.89654	8.26682	9.83202	13.74718	8.13875	8.22483	14.50043	18.55126	14.11218	10.00766	22.66014	28.5095
FF 1052	8.05027	22.72301	23.19289	33.45224	9.17754	17.48721	25.84664	32.52812	11.73816	15.47402	29.79587	35.5672
FFG 7	5.69254	10.28027	17.29269	20.90757	6.52710	7.87229	17.29992	20.09635	8.10780	7.03046	18.42647	21.3237
CG 26	10.24266	28.81892	23.18372	38.37874	9.95373	20.02348	23.48951	32.43105	10.80124	15.45266	24.73446	31.1006
CGN 36	10.70501	28.68997	24.36542	39.13292	10.40151	18.85876	23.85433	32.13835	11.19501	13.95014	23.99435	29.9276
CGN 38	11.61110	30.70165	24.13580	40.74243	11.25916	19.50475	23.82419	32.78408	12.04136	14.04095	23.98830	30.2916
DDG 51	10.59566	24.40950	24.78054	36.36161	11.89454	16.53548	26.23971	33.21783	14.53902	13.38845	28.50159	34.6839
AGF 11	10.96914	26.12556	26.44635	38.75921	10.68594	17.18441	23.95080	31.35497	11.40540	12.80556	23.01104	28.6980
AOR 1	10.75432	26.64081	27.69952	39.90804	10.52486	17.65674	25.27597	32.57925	11.33742	13.21687	24.18918	29.8050
LPD 1	9.97996	21.94000	24.42640	34.31635	9.74951	14.16877	21.64364	27.64515	10.37188	10.45670	20.31884	25.0953
LHD 1	5.87189	17.30922	16.29754	24.48874	4.92679	10.26541	12.81042	17.13939	4.44899	6.76327	10.89827	13.5760
DD 963	10.19566	20.70270	21.87722	31.79884	9.86939	11.67651	19.93022	25.11890	10.30980	8.11578	18.39984	22.5989
DDG 993	10.52465	21.05548	21.80088	32.08395	10.18112	11.65512	19.88580	25.19805	10.60437	8.03969	18.35300	22.6698
LCC 19	11.34512	22.47298	25.73536	36.00077	11.00490	13.01600	22.20399	27.99180	11.50014	9.17032	20.16492	24.9594
CG 47	10.52465	20.34135	21.74536	31.58163	10.17961	11.11982	19.70975	24.81430	10.57916	7.71050	18.10493	22.3419
AOE 1	8.00097	26.76362	20.97783	34.93388	6.62292	14.66127	17.03394	23.43013	5.91194	8.95339	14.35551	17.9219
AD 41	11.25063	22.13919	24.90189	35.16851	10.83201	12.61072	21.94756	27.53286	11.24906	8.87810	20.02214	24.6221
CGN 9	13.78507	28.10387	26.09153	40.75075	13.36414	14.72944	23.85609	31.05913	13.90919	9.97948	22.02665	27.8968
LHA 1	6.98349	22.00052	20.91357	31.14754	5.87255	11.98774	16.31997	21.08398	5.32335	7.37431	13.25322	16.0738
CV 59	7.74030	19.03016	28.06852	34.78363	6.82445	9.78928	18.56518	22.06964	6.35473	6.16980	13.79060	16.3899
CVN 68	8.79649	21.70374	28.11659	36.59198	7.60941	10.59965	19.01722	23.06318	6.95595	6.48964	14.26610	17.1471

RECTILINEAR AMPLITUDES, LOADING FACTORS & SHIP SPEED FACTOR OPERATING CONDITION: **Sea State 6**

Vessel	Sea State	PEAK RECTILINEAR AMPLITUDE				LOADING FACTORS				SPEED FACTORS	
		Long Amplitude	Transv Amplitude	Vertical Amplitude	Combined Amplitude	Long Ldg Fac (ft/s/s)	Transv Ldg Fac (ft/s/s)	Vertical Ldg Fac (ft/s/s)	Combined Ldg Fac (ft/s/s)	Ships Top Speed V (knots)	Combined Vel at Top Speed
T-AGOS 1	4	3.345397	9.106324	8.937902	13.19102	8.92468	12.11942	46.06101	48.45777	11	26.7486
FF 1052	6	8.695894	33.91061	34.77497	49.34416	10.32484	27.87023	54.37589	61.9685	27	62.6853
FFG 7	6	6.183971	18.00176	24.95432	31.38508	7.57021	18.29818	48.14261	52.0561	29	58.2247
CG 26	6	12.56312	43.56339	39.20546	59.93888	11.15760	28.80334	51.60477	60.1430	33	75.6049
CGN 36	6	13.18926	46.66884	42.05255	64.18999	11.59139	27.63256	52.40057	60.3634	31	73.4109
CGN 38	6	14.19963	51.97952	43.60375	69.31657	12.38187	28.26666	52.11554	60.5669	31	74.9169
DDG 51	6	11.29152	42.00241	38.90311	58.35369	13.00791	25.28753	55.60438	62.4541	32	73.3740
AGF 11	6	13.74077	45.03828	45.61529	65.55921	11.56157	25.41976	53.13093	60.0227	21	59.4652
AOR 1	6	13.54257	44.4585	46.18684	65.52239	11.53441	25.89473	54.55804	61.4830	20	58.8062
LPD 1	6	12.51948	39.20601	41.91202	58.74067	10.66595	22.41629	51.48171	57.1543	22	57.4240
LHD 1	6	8.15564	33.23082	31.8899	46.77358	5.61612	17.47830	43.20458	46.9432	23	50.6214
DD 963	6	12.5327	43.40111	39.54362	60.03684	11.13309	20.45700	50.29727	55.4279	32	70.9192
DDG 993	6	12.89921	45.35588	40.10868	61.90519	11.42839	20.43128	50.24011	55.4267	31	69.7731
LCC 19	6	14.22956	47.48223	47.52071	68.66775	11.95538	21.24672	52.48152	57.8676	21	57.9357
CG 47	6	12.9047	45.38512	40.2134	61.99563	11.43280	19.89599	50.21694	55.2114	31	69.5436
AOE 1	6	11.09585	52.41942	42.57157	68.43428	7.44444	21.86092	46.82596	52.2110	28	64.8842
AD 41	6	13.9968	47.10662	45.15725	66.7392	12.02186	20.84260	52.35564	57.6199	20	55.9986
CGN 9	6	16.87248	65.88377	52.02724	85.62818	14.42930	23.46282	53.68465	60.3386	32	77.8891
LHA 1	6	9.811008	44.09368	40.92715	60.95523	6.65678	19.19372	46.75754	50.9802	24	56.3628
CV 59	6	11.57454	51.33522	64.31762	83.10253	6.94339	16.99055	49.80348	53.0780	33	71.9328
CVN 68	6	12.89708	60.2664	65.78852	90.14712	7.82271	17.79435	50.09100	53.7303	33	73.6278

OPERATING CONDITION:

SHIP MOTION CHARACTERISTICS

SHIP MOTION INTERFACE CHARACTERISTICS			
PERFORMANCE CHARACTERISTIC	ROLL (φ)	PITCH (θ)	COMB (φ & θ)
MAXIMUM ANGULAR AMPLITUDE (degrees)	19.000	3.000	19.235
MAXIMUM ANGULAR VELOCITY (deg/sec)	11.541	3.142	11.961
MAXIMUM ANGULAR ACCELERATION (deg/s/s)	7.011	4.935	7.744
	LONG (x)	TRANSV (y)	VERT (z)
MAXIMUM RECTILINEAR AMPLITUDE (feet)	16.872	65.884	65.789
MAXIMUM RECTILINEAR VELOCITY (ft/sec)	13.785	30.702	28.117
MAXIMUM RECTILINEAR ACCELERATION (ft/s/s)	13.364	20.023	26.240
MAXIMUM RECTILINEAR JERK (ft/s/s/s)	14.539	15.474	29.796
MAX VELOCITY AT TOP SPEED (ft/sec)			77.889

PLATFORM MOTION CHARACTERISTICS

MOTION CHARACTERISTIC	AMPLITUDE	PERIOD
PITCH	2.000 deg	4.000 sec
ROLL	19.000 deg	10.344 sec
YAW	1.000 deg	4.000 sec
SURGE	13.423 ft	6.118 sec
SWAY	43.246 ft	8.850 sec
HEAVE	27.313 ft	6.104 sec
COMBINED ROTATION	18.475 deg	9.705 sec
COMBINED TRANSLATION	43.619 ft	6.725 sec

MAXIMUM LOADING FACTOR COMPONENTS

LONGITUDINAL LDG FACTOR	Ax =	14.429 ft/sec/sec	(0.449g)
TRANSVERSE LDG FACTOR	Ay =	28.803 ft/sec/sec	(0.895g)
VERTICAL LOADING FACTOR	Az =	55.604 ft/sec/sec	(1.728g)
COMBINED LDG FACTOR	Ac =	62.454 ft/sec/sec	(1.941g)

DOD-STD-1399, Sec 301A, Table II

Beam Range (ft)	Roll Angle (degrees)							
	Sea State							
	4	5	6	7	8	7	8	8
0	7	12	19	28	42	28	42	42
50	6	10	16	24	37	24	37	37
75	6	10	15	22	34	22	34	34
105	5	9	13	20	31	20	31	31
greater								

BOOK TABLES

DOD-STD-1399, Sec 301A, Table III

LBP Range (ft)	Pitch Angle (degrees)								Pitch Period (seconds)
	Sea State								
	4	5	6	7	8	7	8	8	Sea States 4 - 8
0	2	3	5	7	11	7	11	11	
150	2	3	4	6	10	6	10	10	
250	1	2	4	6	9	6	9	9	
350	1	2	3	5	7	5	7	7	
500	1	2	3	4	6	4	6	6	
700	1	1	2	3	5	3	5	5	
greater									

DOD-STD-1399, Sec 301A, Table IV

LBP Range (ft)	Heave Acceleration (g)							
	Sea State							
	4	5	6	7	8	7	8	8
0	0.1	0.17	0.27	0.4	0.6	0.4	0.6	0.6
150	0.1	0.17	0.27	0.4	0.6	0.4	0.6	0.6
250	0.1	0.17	0.27	0.4	0.6	0.4	0.6	0.6
350	0.08	0.14	0.21	0.3	0.5	0.3	0.5	0.5
500	0.06	0.1	0.16	0.2	0.4	0.2	0.4	0.4
700	0.04	0.07	0.11	0.2	0.2	0.2	0.2	0.2
greater								

DOD-STD-1399, Sec 301A, Table V

LBP Range (ft)	Surge Acceleration (g)							
	Sea State							
	4	5	6	7	8	7	8	8
0	0.06	0.1	0.15	0.25	0.35	0.25	0.35	0.35
150	0.05	0.1	0.15	0.2	0.3	0.2	0.3	0.3
250	0.05	0.1	0.15	0.2	0.3	0.2	0.3	0.3
350	0.04	0.05	0.1	0.15	0.25	0.15	0.25	0.25
500	0.04	0.05	0.1	0.15	0.25	0.15	0.25	0.25
700	0.02	0.05	0.05	0.1	0.1	0.15	0.15	0.15
greater								

APPENDIX B
SHIP-MOTION PERFORMANCE-CHARACTERISTICS
COMPUTATIONAL METHODS

INTRODUCTION

NOMENCLATURE

- ϕ = Maximum roll angle (radians)
- θ = Maximum pitch angle (radians)
- μ = Maximum yaw angle (radians)
- δ_x = Maximum surge amplitude (feet)
- δ_y = Maximum sway amplitude (feet)
- δ_z = Maximum heave amplitude (feet)
- T_r = Modal roll period (seconds)
- T_p = Modal pitch period (seconds)
- T_μ = Modal yaw period (seconds)
- T_x = Modal surge period (seconds)
- T_y = Modal sway period (seconds)
- T_z = Modal heave period (seconds)
- s = Peak surge acceleration (ft/sec²)
- h = Peak heave acceleration (ft/sec²)
- x = Distance forward of ship's mean cg location in inertial reference system (ft)
- y = Distance port of ship's mean cg location in inertial reference system (ft)
- z = Distance above ship's mean cg location in inertial reference system (ft)
- X = Longitudinal distance forward of ship's cg in ship's reference system (ft)
- Y = Transverse distance port of ship's cg in ship's reference system (ft)
- Z = Vertical distance above ship's cg (ft) in ship's reference system (ft)
- Ω = Maximum angular displacement (radians)
- ω = Maximum angular velocity (rad/sec)
- α = Maximum angular acceleration (rad/sec²)
- d = Maximum rectilinear displacement (ft)
- v = Maximum rectilinear velocity (ft/sec)
- a = Maximum rectilinear acceleration (ft/sec²)
- j = Maximum rectilinear jerk (ft/sec³)
- t = time (seconds)

Subscripts and diacritical notation

r = roll

p = pitch

P = Point P stationary in ship's reference system

Q = Point Q stationary in inertial reference system

This analysis deals largely with estimating and manipulating peak values of motion variables. Consequently, representing maximum values and constants in the shortest possible (i.e., single character) format is convenient. Where needed to represent these parameters as variables, they will be displayed in function format, that is, with parentheses. An exception to this is the variable time, which is represented by lowercase t . For example,

θ	= maximum pitch amplitude;	$\theta(t)$	= pitch angle (variable)
$\ddot{\delta}_x$	= max surge acceleration;	$\ddot{\delta}_x(t)$	= surge acceleration (var)
T_r	= roll period (constant);	t	= time (variable)

REFERENCE SYSTEMS

Two Cartesian coordinate systems are used to develop the equations describing a ship's motion in rough seas. The first system, denoted by axes x , y , and z , is a nonaccelerating inertial reference system, needed for resolving dynamic forces by applying Newton's second law. Its origin is assumed to be located at the mean position of the ship's center of gravity (cg) and to move at a constant velocity equal to—and in the same direction as—the ship's mean forward speed. Its orientation is such that the x -axis is aligned with the ship's mean forward direction and the z -axis is vertical (perpendicular to the sea surface). Accelerations of the system due to changes in speed and heading—or to curvature and rotation of the earth—are small when compared with those of the ship in rough seas; therefore, they are considered negligible.

The second coordinate system, denoted by axes X , Y , and Z , is affixed to the ship and follows the ship's irregular motion about the first system. Its origin is located at the ship's center of gravity, and the X , Y , and Z axes are aligned in the forward, port, and vertical (perpendicular to the deck) directions, respectively. This reference system is used to define shipboard locations and to simplify mathematical expression of ship motion.

Figure B-1 depicts the two reference systems. The ship's reference system translates and rotates with respect to the inertial system due to the irregular motion of the seas. This relative motion may be partitioned into mutually exclusive components that conform with the six possible degrees-of-freedom. These six components correspond with the six conventional components of ship motion: surge, sway, heave, roll, pitch, and yaw.

The directions of the six components of ship motion are conventionally assumed to align with the axes of the ship's reference system. To simplify computation, assume that surge, sway, and heave motions are in the directions of the x , y , and z axes of the inertial reference system. This assumption greatly reduces the amount of computation and bookkeeping required, and is sound, because the methods used in this analysis enable complete separation of translational and rotational motion. The final results are the same, if, when the translational components are added into the overall equations of motion, the coupling angles are set at zero.

EQUATIONS OF MOTION

For estimating purposes, the six components of ship motion may be assumed to be oscillatory (sinusoidal). Since only realistic worst-case values are of interest, each of these may be characterized by a peak amplitude and a modal period. A maximum-displacement situation occurs when the six components of motion peak simultaneously. By assuming this event occurs at time, $t = 0$, the phase angles may be eliminated and the component equations are simplified:

$$\begin{array}{ll} \text{surge, } \delta_x(t) = \delta_x \cos \frac{2\pi}{T_x} t & \text{roll, } \phi(t) = \phi \cos \frac{2\pi}{T_r} t \\ \text{sway, } \delta_y(t) = \delta_y \cos \frac{2\pi}{T_y} t & \text{pitch, } \theta(t) = \theta \cos \frac{2\pi}{T_p} t \\ \text{heave, } \delta_z(t) = \delta_z \cos \frac{2\pi}{T_z} t & \text{yaw, } \mu(t) = \mu \cos \frac{2\pi}{T_\mu} t \end{array}$$

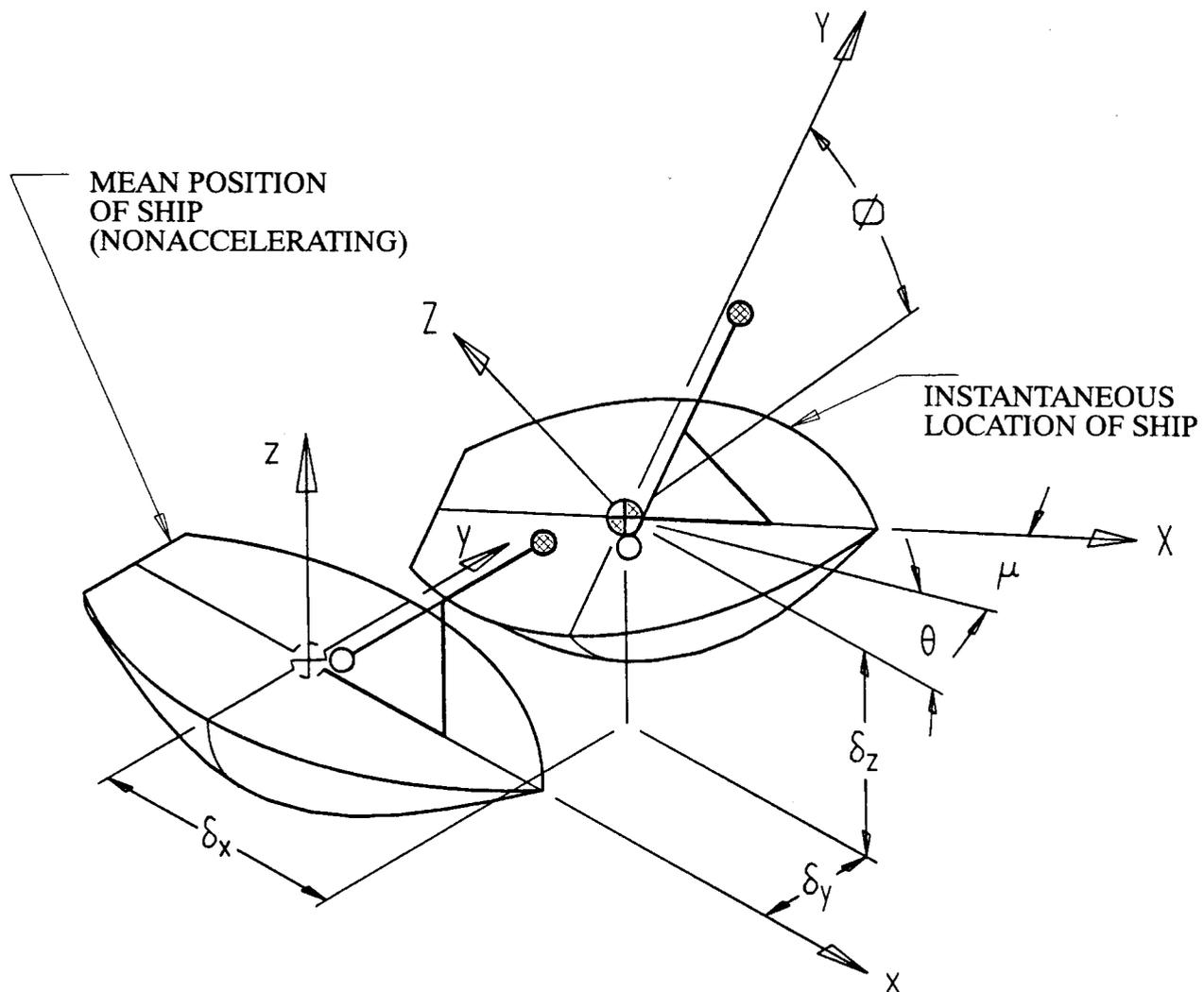


Figure B-1. Inertial and ship's reference systems.

where,

δ_x	=	maximum surge amplitude	T_x	=	modal surge period
δ_y	=	maximum sway amplitude	T_y	=	modal sway period
δ_z	=	maximum heave amplitude	T_z	=	modal heave period
ϕ	=	maximum roll amplitude	T_r	=	modal roll period
θ	=	maximum pitch amplitude	T_p	=	modal pitch period
μ	=	maximum yaw amplitude	T_μ	=	modal yaw period

These equations may be further simplified by adopting several approximations suggested in DoD-STD-1399, Section 301A (reference 10). These approximations are based on observations of empirical data for monohull naval ships. Most notably, yaw and sway motions are small when compared to the other components of ship motion and may therefore be neglected in the development of ship-motion equations. Also noted is that the peak amplitudes and modal periods for the heave and surge components are not very distinct. Heave and surge are best characterized in terms of their peak acceleration values. Heave and surge motions are somewhat coupled to pitch motion and, for all practical purposes, their periods may be assumed to be equal to the modal pitch period. These approximations are expressed mathematically in the following equations:

$$\delta_y = \mu = 0 \text{ (negligible)}$$

$$(T_y \text{ and } T_\mu \text{ disregarded})$$

$$T_x \approx T_z \approx T_p$$

The second derivatives (with respect to time) of the equations for surge and heave provide expressions for surge and heave acceleration. These may be used to provide expressions for the peak surge and heave amplitudes in terms of their respective peak acceleration values:

$$\delta_x \approx \frac{T_p^2}{4\pi^2} s$$

$$\delta_z \approx \frac{T_p^2}{4\pi^2} h$$

where,

$s \equiv$ peak surge acceleration

$h \equiv$ peak heave acceleration

By incorporating these approximations into the six component equations, they may be rewritten:

$$\begin{aligned} \text{surge, } \delta_x(t) &= \frac{T_p^2}{4\pi^2} s \cos \frac{2\pi}{T_p} t & \text{roll, } \phi(t) &= \phi \cos \frac{2\pi}{T_r} t \\ \text{sway, } \delta_y(t) &= 0 & \text{pitch, } \theta(t) &= \theta \cos \frac{2\pi}{T_p} t \\ \text{heave, } \delta_z(t) &= \frac{T_p^2}{4\pi^2} h \cos \frac{2\pi}{T_p} t & \text{yaw, } \mu(t) &= 0 \end{aligned}$$

ANGULAR-MOTION EQUATIONS

The angular motion of the ship with respect to the inertial reference system is independent of location. Estimates of the peak angular velocities and peak angular accelerations due to roll and pitch may be developed by differentiating their respective periodic equations:

Roll velocity,

$$\dot{\phi}(t) = -\frac{2\pi}{T_r} \phi \sin \frac{2\pi}{T_r} t$$

Pitch velocity,

$$\dot{\theta}(t) = -\frac{2\pi}{T_p} \theta \sin \frac{2\pi}{T_p} t$$

Roll acceleration,

$$\ddot{\phi}(t) = -\frac{4\pi^2}{T_r^2} \phi \sin \frac{2\pi}{T_r} t$$

Pitch acceleration,

$$\ddot{\theta}(t) = -\frac{4\pi^2}{T_p^2} \theta \sin \frac{2\pi}{T_p} t$$

Maximum roll velocity,

$$\dot{\phi} = \frac{2\pi}{T_r} \phi$$

Maximum pitch velocity,

$$\dot{\theta} = \frac{2\pi}{T_p} \theta$$

Maximum roll acceleration,

$$\ddot{\phi} = \frac{4\pi^2}{T_r^2} \phi$$

Maximum pitch acceleration,

$$\ddot{\theta} = \frac{4\pi^2}{T_p^2} \theta$$

TRANSFORMATION EQUATIONS

Before equations can be developed to describe the relationship between the two reference systems, the angular components of ship motion must first be defined in terms of the sequence in which they are carried out. A change in this order alters their relative amplitudes, and an inconsistency in applying this sequence results in accumulation of error. The mathematical explanation for this lies in the noncommutative nature of matrix multiplication. The order used throughout this study is yaw-pitch-roll.

The relationship between the three angular components is depicted in figure B-2. The vessel shown is assumed to be positioned so that all six components of motion are at maximum amplitude. The three views shown are projected so each displays one of the angular components in its true perspective. An arbitrary point on the vessel, P, is shown in each of the three views. Its coordinates in the inertial reference system and the ship's reference system are shown. The relationship between the inertial reference system and the ship's reference system may be discerned from the graphic projections in figure B-2:

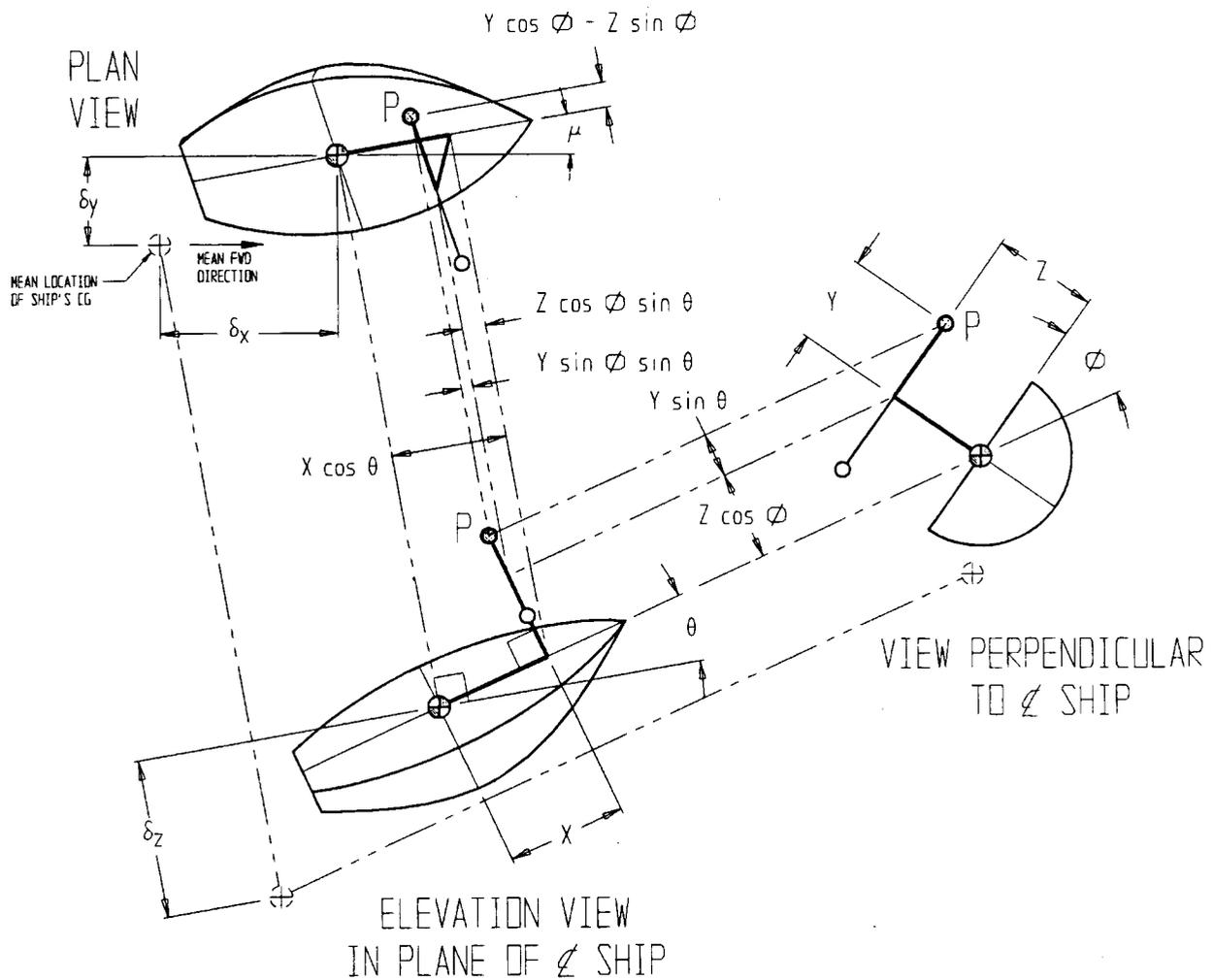


Figure B-2. True projection of angular components of motion.

$$\begin{aligned}
x &= \delta_x + X \cos \theta \cos \mu - Y \sin \phi \sin \theta \cos \mu - Y \cos \phi \sin \mu \\
&\quad - Z \cos \phi \sin \theta \cos \mu + Z \sin \phi \sin \mu \\
y &= \delta_y + X \cos \theta \sin \mu - Y \sin \phi \sin \theta \sin \mu + Y \cos \phi \cos \mu \\
&\quad - Z \cos \phi \sin \theta \sin \mu + Z \sin \phi \cos \mu \\
z &= \delta_z + X \sin \theta + Y \sin \phi \cos \theta + Z \cos \phi \cos \theta
\end{aligned}$$

These three equations describe the vector location of the point, P, in the inertial reference system. By putting these equations in a convenient matrix format and applying matrix-inversion techniques, the inverse equations are developed. That is,

$$[x - \delta] = [A] [X]$$

$$[X] = [A^{-1}] [x - \delta]$$

The new set of equations describes the displacement of the outside world relative to the ship at point P due to ship motion. This perspective is used, for example, to develop expressions for the motion of stationary satellites relative to a shipboard antenna. Note that the constant forward speed of the ship is not considered in these equations, since the inertial reference system is assumed to track the mean position of the ship's cg. The transposed equations are

$$X = (x - \delta_x) (\cos \theta \cos \mu) + (y - \delta_y) (\cos \theta \sin \mu) + (z - \delta_z) (\sin \theta)$$

$$\begin{aligned}
Y &= (x - \delta_x) (-\sin \phi \sin \theta \cos \mu - \cos \phi \sin \mu) \\
&\quad + (y - \delta_y) (-\sin \phi \sin \theta \sin \mu + \cos \phi \cos \mu) + (z - \delta_z) (\sin \phi \cos \theta)
\end{aligned}$$

$$\begin{aligned}
Z &= (x - \delta_x) (-\cos \phi \sin \theta \cos \mu + \sin \phi \sin \mu) \\
&\quad + (y - \delta_y) (-\cos \phi \sin \theta \sin \mu - \sin \phi \cos \mu) + (z - \delta_z) (\cos \phi \cos \theta)
\end{aligned}$$

These two sets of transformation equations reflect the static relationship between the two reference systems at a given instant ($t = 0$). Motion is not considered in their development. Expressions for the velocity and acceleration of a point in one system, relative to the other, may be determined by taking time derivatives of its position vector. First, however, all time variables, including the six component amplitudes, must be expressed as variables. Adopting the assumptions and approximations regarding component motion, the two sets of transform equations may be written in general form.

$$x(t) - \delta_x(t) = X(t) \cos \theta(t) - Y(t) \sin \phi(t) \sin \theta(t) - Z(t) \cos \phi(t) \sin \theta(t)$$

$$y(t) - \delta_y(t) = Y(t) \cos \phi(t) + Z(t) \sin \phi(t)$$

$$z(t) - \delta_z(t) = X(t) \sin \theta(t) + Y(t) \sin \phi(t) \cos \theta(t) + Z(t) \cos \phi(t) \cos \theta(t)$$

$$X(t) = [x(t) - \delta_x(t)] \cos \theta(t) + [z(t) - \delta_z(t)] \sin \theta(t)$$

$$Y(t) = [x(t) - \delta_x(t)] [-\sin \phi(t) \sin \theta(t)] + y(t) \cos \phi(t) + [z(t) - \delta_z(t)] [\sin \phi(t) \cos \theta(t)]$$

$$Z(t) = [x(t) - \delta_x(t)] [-\cos \phi(t) \sin \theta(t)] - y(t) \sin \phi(t) + [z(t) - \delta_z(t)] [\cos \phi(t) \cos \theta(t)]$$

PERFORMANCE PERSPECTIVE

The velocities, accelerations, and other motion attributes that comprise the performance characteristics are defined from the perspective of a piece of equipment affixed to the ship. From the point of view of this equipment, the locations of all points in space are perceived in the ship's coordinate system. The three transformation equations defining $X(t)$, $Y(t)$, and $Z(t)$ describe the location of such a point in terms of its coordinates in the inertial reference system. The velocity of this point, from the equipment's perspective, may be found by taking the derivative, with respect to time, of the three equations, assuming that its motion in the inertial reference system can be described. Likewise, the acceleration and jerk of this point are described by taking the second and third derivatives, respectively. To illustrate this, the X -component of the position, velocity, and acceleration equations for a point Q are developed here.

$$\begin{aligned}
 X_Q(t) &= [x_Q(t) - \delta_{x_Q}(t)] \cos \theta(t) + [z_Q(t) - \delta_{z_Q}(t)] \sin \theta(t) \\
 \dot{X}_Q(t) &= [\dot{x}_Q(t) - \dot{\delta}_{x_Q}(t)] \cos \theta(t) - [x_Q(t) - \delta_{x_Q}(t)] \dot{\theta} \sin \theta(t) \\
 &\quad + [\dot{z}_Q(t) - \dot{\delta}_{z_Q}(t)] \sin \theta(t) + [z_Q(t) - \delta_{z_Q}(t)] \dot{\theta} \cos \theta(t) \\
 \ddot{X}_Q(t) &= [\ddot{x}_Q(t) - \ddot{\delta}_{x_Q}(t)] \cos \theta(t) - 2[\dot{x}_Q(t) - \dot{\delta}_{x_Q}(t)] \dot{\theta} \sin \theta(t) \\
 &\quad - [x_Q(t) - \delta_{x_Q}(t)] \ddot{\theta} \sin \theta(t) - [x_Q(t) - \delta_{x_Q}(t)] \dot{\theta}^2 \cos \theta(t) \\
 &\quad + [\ddot{z}_Q(t) - \ddot{\delta}_{z_Q}(t)] \sin \theta(t) + 2[\dot{z}_Q(t) - \dot{\delta}_{z_Q}(t)] \dot{\theta} \cos \theta(t) \\
 &\quad + [z_Q(t) - \delta_{z_Q}(t)] \ddot{\theta} \cos \theta(t) - [z_Q(t) - \delta_{z_Q}(t)] \dot{\theta}^2 \sin \theta(t)
 \end{aligned}$$

If the equipment is located at a point, P , affixed to the ship, its location in the ship's reference system may be given by the coordinates (X_P, Y_P, Z_P) , where X_P , Y_P and Z_P are constants. At any moment in time, the equipment's perspective of the inertial system's relative motion is fully represented by the point in inertial space that happens to coincide with point P at that instant. This perspective provides the desired vantage point for defining performance characteristics. Point Q is here defined as that point fixed in inertial space that at any instant occupies the same location as the observation point on the ship, P . In the inertial system, its location may be described by the equations,

$$\begin{aligned}
 [x_Q(t) - \delta_{x_Q}(t)] &= x_P(t) - \delta_{x_P}(t) = X_P \cos \theta(t) - Y_P \sin \phi(t) \sin \theta(t) - Z_P \cos \phi(t) \sin \theta(t) \\
 [y_Q(t)] &= y_P(t) = Y_P \cos \phi(t) + Z_P \sin \phi(t) \\
 [z_Q(t) - \delta_{z_Q}(t)] &= z_P(t) - \delta_{z_P}(t) = X_P \sin \theta(t) + Y_P \sin \phi(t) \cos \theta(t) + Z_P \cos \phi(t) \cos \theta(t)
 \end{aligned}$$

Since point Q is fixed in inertial space, it has no velocity or acceleration in the inertial reference frame.

$$\dot{x}_Q(t) = \dot{y}_Q(t) = \dot{z}_Q(t) = \ddot{x}_Q(t) = \ddot{y}_Q(t) = \ddot{z}_Q(t) = 0$$

$$[\dot{x}_Q(t) - \dot{\delta}_{x_Q}(t)] = \frac{T_p}{2\pi} s \sin \frac{2\pi}{T_p} t \quad [\ddot{x}_Q(t) - \ddot{\delta}_{x_Q}(t)] = s \sin \frac{2\pi}{T_p} t$$

$$[\dot{z}_Q(t) - \dot{\delta}_{z_Q}(t)] = \frac{T_p}{2\pi} h \sin \frac{2\pi}{T_p} t \quad [\ddot{z}_Q(t) - \ddot{\delta}_{z_Q}(t)] = h \sin \frac{2\pi}{T_p} t$$

These equations accommodate substitution of the bracketed terms in the equations for position, velocity, and acceleration at point Q. Time variables are set for maximum amplitude. Note that the substitution of heave and surge terms into the equations is made at zero roll and pitch angles.

$$X_Q(t) = X_p$$

$$\dot{X}_Q(t) = \frac{T_p}{2\pi} s + \dot{\theta} Y_p \sin \phi(t) + \dot{\theta} Z_p \cos \phi(t)$$

$$\ddot{X}_Q(t) = s - \ddot{\theta}^2 X_p + \ddot{\theta} Y_p \sin \phi(t) + \ddot{\theta} Z_p \cos \phi(t)$$

No deflection is computed for point Q since it is continuously redefined at the same point. The deflection of the points having occupied this position has no significance to the performance characteristics being considered. Later, however, knowing their relative amplitudes might be desirable. The excursion, or relative deflection, at this point, is computed by first determining the travel of the point in the inertial reference system. For point Q,

$$\Delta[x_Q - \delta_{x_Q}] = \frac{T_p^2}{4\pi^2} s + Y_p \sin \phi \sin \theta + Z_p \cos \phi \sin \theta$$

$$\Delta y_Q = Z_p \sin \phi$$

$$\Delta[z_Q - \delta_{z_Q}] = \frac{T_p^2}{4\pi^2} h + X_p \sin \theta + Y_p \sin \phi \cos \theta$$

Using the transformation equations,

$$\Delta X_Q = \frac{T_p^2}{4\pi^2} s + X_p \sin^2 \theta + 2 Y_p \sin \phi \sin \theta \cos \theta + Z_p \cos \phi \sin \theta \cos \theta$$

The maximum value of each of the components is determined using trial and error methods, since the range of roll and pitch angles is limited. Signs are adjusted to generate the maximum value of each expression. The same approach is used to develop the equations for the transverse and vertical components of motion. The combined deflections, velocities, accelerations, and jerks are estimated by computing the square root of the sum of the squares of the directional components. To further simplify mathematical expression of the performance characteristics, small angles are assumed,

$$\sin \phi \approx \phi \quad \cos \phi \approx 1$$

$$\sin \theta \approx \theta \quad \cos \theta \approx 1$$

PERFORMANCE CHARACTERISTICS

The equations developed for the peak value components of ship motion are listed here. These equations comprise the performance characteristics used in this study.

ANGULAR DISPLACEMENT COMPONENTS

Maximum roll angle (radians)

$$\phi$$

Maximum pitch angle (radians)

$$\theta$$

Maximum combined angular displacement (radians)

$$\Omega = \sqrt{\phi^2 + \theta^2}$$

ANGULAR VELOCITY COMPONENTS

Maximum roll angular velocity (rad/sec)

$$\dot{\phi} = \frac{2\pi}{T_r} \phi$$

Maximum pitch angular velocity (rad/sec)

$$\dot{\theta} = \frac{2\pi}{T_p} \theta$$

Maximum combined angular velocity (rad/sec)

$$\omega = \sqrt{\dot{\phi}^2 + \dot{\theta}^2}$$

ANGULAR ACCELERATION COMPONENTS

Maximum roll angular acceleration (rad/sec²)

$$\ddot{\phi} = \frac{4\pi^2}{T_r^2} \phi$$

Maximum pitch angular acceleration (rad/sec²)

$$\ddot{\theta} = \frac{4\pi^2}{T_p^2} \theta$$

Maximum combined angular acceleration (rad/sec²)

$$\alpha = \sqrt{\ddot{\phi}^2 + \ddot{\theta}^2}$$

RELATIVE DISPLACEMENT COMPONENTS

Maximum relative longitudinal displacement (excursion)(ft)

$$\Delta X_Q = \frac{T_p^2}{4\pi^2} s + \theta^2 X_P + 2 \theta \phi Y_P + \theta Z_P$$

Maximum relative transverse displacement (excursion) (ft)

$$\Delta Y_Q = \theta \phi X_P + \phi^2 Y_P + \phi Z_P$$

Maximum relative vertical displacement (excursion) (ft)

$$\Delta Z_Q = \frac{T_p^2}{4\pi^2} h + \theta X_P + \phi Y_P + (\phi^2 + \theta^2) Z_P$$

Maximum combined rectilinear relative displacement (excursion) (ft)

$$\Delta d_Q = \sqrt{\Delta X_Q^2 + \Delta Y_Q^2 + \Delta Z_Q^2}$$

RECTILINEAR VELOCITY COMPONENTS

Maximum longitudinal velocity (ft/sec)

$$\dot{X}_Q = \frac{T_p}{2\pi} s + \frac{2\pi}{T_p} \theta \phi Y_P + \frac{2\pi}{T_p} \theta Z_P$$

Maximum transverse velocity (ft/sec)

$$\dot{Y}_Q = \frac{1}{2} \cdot \frac{2\pi}{T_p} \theta X_P + \frac{2\pi}{T_p} \phi Z_P$$

Maximum vertical velocity (ft/sec)

$$\dot{Z}_Q = \frac{T_p}{2\pi} h + \frac{2\pi}{T_p} \phi X_P + \frac{2\pi}{T_p} \phi Y_P$$

Maximum rectilinear velocity (ft/sec)

$$v_Q = \sqrt{\dot{X}_Q^2 + \dot{Y}_Q^2 + \dot{Z}_Q^2}$$

RECTILINEAR ACCELERATION COMPONENTS

Maximum longitudinal acceleration (ft/sec²)

$$\ddot{X}_Q = s + \frac{4\pi^2}{T_p^2} \theta^2 X_P + \frac{4\pi^2}{T_p^2} \phi \theta Y_P + \frac{8\pi^2}{T_p T_r} \phi \theta (Y_P - \phi Z_P) + \frac{4\pi^2}{T_p^2} \theta Z_P$$

Maximum transverse acceleration (ft/sec²)

$$\ddot{Y}_Q = \frac{1}{2} \cdot \frac{4\pi^2}{T_p^2} \theta X_P + \frac{4\pi^2}{T_r^2} \phi^2 Y_P + \frac{4\pi^2}{T_r^2} \phi Z_P + \frac{4\pi^2}{T_p^2} \phi \theta^2 (\phi Y_P + Z_P)$$

Maximum transverse acceleration (ft/sec²)

$$\ddot{Z}_Q = h + \frac{4\pi^2}{T_p^2} \theta X_P + \frac{4\pi^2}{T_r^2} \phi Y_P + \frac{4\pi^2}{T_r^2} \phi^2 Z_P + \frac{4\pi^2}{T_p^2} \theta^2 (\phi Y_P + Z_P)$$

Maximum transverse acceleration (ft/sec²)

$$a_Q = \sqrt{\ddot{X}_Q^2 + \ddot{Y}_Q^2 + \ddot{Z}_Q^2}$$

RECTILINEAR JERK COMPONENTS

Maximum longitudinal jerk (ft/sec³)

$$\begin{aligned} \ddot{\dot{X}}_Q = & \frac{2\pi}{T_p} s + \frac{24\pi^3}{T_p^3} \theta^2 X_P + \frac{24\pi^3}{T_p^2 T_r} \phi \theta (Y_P + \phi Z_P) + \frac{24\pi^3}{T_p T_r^2} \phi \theta Y_P (1 + \phi^2) \\ & + \frac{8\pi^2}{T_p^3} \theta (\phi Y_P + Z_P) (1 + \phi^2) \end{aligned}$$

Maximum transverse jerk (ft/sec³)

$$\begin{aligned} \ddot{\dot{Y}}_Q = & \frac{1}{2} \cdot \frac{8\pi^3}{T_p^3} \theta X_P (1 + \theta^2) + \frac{24\pi^3}{T_r^3} \phi^2 Y_P + \frac{8\pi^2}{T_r^3} \phi Z_P (1 + \phi^2) + \\ & + \frac{24\pi^3}{T_p^2} \theta^2 \left[\frac{\phi^2}{T_r} (Y_P - \phi Z_P) + \frac{\phi}{T_p} (\phi Y_P + Z_P) \right] \end{aligned}$$

Maximum vertical jerk (ft/sec³)

$$\begin{aligned}\ddot{Z}_Q &= \frac{2\pi}{T_p} h + \frac{8\pi^3}{T_p^3} \theta X_P(1 + \theta^2) + \frac{8\pi^3}{T_r^3} \phi Y_P(1 + \phi^2) + \frac{24\pi^3}{T_p^3} \theta^2(\phi Y_P + Z_P) \\ &+ \frac{24\pi^3}{T_r^3} \phi^2(\phi Y_P + Z_P) + \frac{24\pi^3}{T_p^2 T_r} \theta^2 \phi(Y_P - \phi Z_P)\end{aligned}$$

Maximum rectilinear jerk (ft/sec³)

$$j_Q = \sqrt{\ddot{X}_Q^2 + \ddot{Y}_Q^2 + \ddot{Z}_Q^2}$$